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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 123 MAY, 1921

STUDIES ON COOLING OF FRESH CONCRETE IN FREEZING WEATHER

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ENGINEERING EXPERIMENT STATION
PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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STUDIES ON COOLING OF FRESH CONCRETE IN FREEZING WEATHER *

I. Introduction

1. *Preliminary.*—The practice of placing concrete in freezing weather renders important a knowledge of the rate at which this material will cool and the effect of the various methods of protecting the freshly placed concrete from the cold. The experiments herein recorded furnish test data on the length of time required for concrete of a given temperature to lose its heat and become cold enough to freeze when it is exposed to temperatures lower than the freezing point of water. The values of two thermal constants, diffusivity and the ratio of emissivity to coefficient of thermal conductivity, were determined for freshly placed concrete. Some experiments were made on the protective effects of coverings. While a number of applications of the experimental data are presented, no originality can be claimed for the underlying mathematical theory, which dates back, of course, to the time of Fourier. It is not considered that these solutions will give complete data regarding concreting in freezing weather under various conditions; however, it is hoped that they will throw some light on the behavior of fresh concrete at low temperatures and will indicate the necessity of protection or other precautions.

2. *Acknowledgments.*—The tests reported herein were made in the Engineering Experiment Station of the University of Illinois. The writer is indebted to Professor A. N. Talbot, Professor H. F. Gonnerman, Mr. F. E. Richart, Mr. R. A. Nelson, and Mr. W. H. Braman.

3. *Discussion Regarding Assumptions.*—Every calculation in engineering is based upon some assumption. This is not strange when we consider that the principles and laws of physics and chemistry upon which engineering is founded are based upon fundamental hypotheses or principles.

* This bulletin presents the principal results of the investigation made in 1920 on the flow of heat through fresh concrete by Tokujiro Yoshida, Assistant Professor of Civil Engineering, Kyushu Imperial University, Fukuoka, Japan, who was in residence as a graduate student at the University of Illinois.

Mathematicians may be satisfied by proving that the solution of a problem is possible, but the engineer cannot be satisfied until he obtains the particular numerical value which is applicable to an actual case. In this fact is found the reason for the difference between the view points of physicists or mathematicians and engineers.

Engineering problems are usually very complicated. There are many conditions which cannot be expressed mathematically, or which are entirely unknown. So, if the results obtained by physicists or mathematicians are to be applied to engineering, many assumptions must be made, on the one hand, in order that the conditions may be expressed physically or mathematically, while, on the other hand, the calculations must be made as simple as possible for actual use. In general, the nearer the assumption is to the actual fact, the more accurate will be the result based on the assumption; but no matter how accurate the result, if the time spent in calculation is too long the method is often not acceptable to the engineer. When a method of calculation or an engineering formula is used, the assumptions on which it is based must first be known, and then these assumptions compared with the actual conditions affecting the case under consideration. It is an important function of engineering science to conduct researches into conditions affecting engineering problems, in order that assumptions may be made which will approximate the facts of the case and at the same time render possible the introduction of methods of calculation that are reasonably simple.

In the following problems a number of assumptions have been made in order to simplify calculations as well as to cover many conditions that are unknown. It is important that this discussion with reference to assumptions be borne in mind by the reader.

II. Experiments on Cooling of Fresh Concrete and Rise in Temperature during Setting of Concrete and Mortar

4. *Apparatus for Measuring Temperature.*—The general scheme of the experiments on the cooling of fresh concrete involved the exposing of one surface of a specimen to a low temperature, all other surfaces being carefully insulated from the cold, and the measuring of the temperature of the concrete at various distances from the exposed surface. After a study of several methods for the accurate measurement of these temperatures, a system of thermocouples was chosen as most suitable for the purpose. Fig. 1 shows the general layout of the apparatus for measuring temperature.

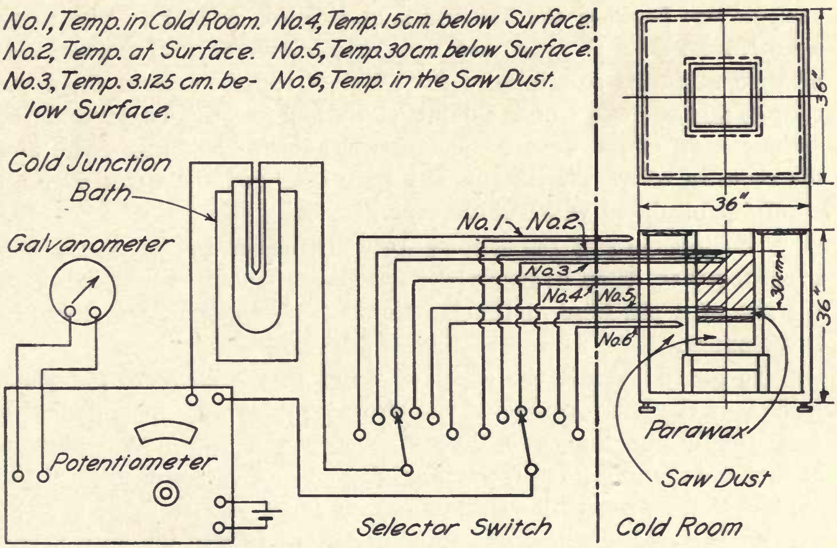

Fig. 1. Layout of Temperature-Measuring Apparatus

By the use of a number of thermocouples connected to a sensitive potentiometer the temperatures at various points were quickly and accurately indicated in terms of electrical units, which were later converted into degrees centigrade for use in making computations. Since the range of temperatures to be measured in this investigation was relatively small, and hence the electromotive force in the ther-

mocouple circuit also small, the use of the deflection method was considered inadvisable, because in this method the readings are not independent of the resistance in the circuit and are liable to errors due to changes in resistance. The potentiometer was found to be sufficiently accurate for the purpose in view, and since the principle involved embodied the measurement of the electromotive force with zero current in the circuit, thus making the readings independent of the external resistance, it was adopted. In the potentiometer method the precision of reading is a function of the accuracy with which the initial balance is attained. The accuracy of the initial balance was greatly increased in the present case by using a sensitive galvanometer.

The wires used in the thermocouple were of copper and of a copper-nickel alloy known under the trade name of constantan. These metals were chosen because they are homogeneous and because the magnitude of the induced electromotive force per degree difference in temperature between the hot and cold junction is large. The wires were No. 20 B. & S. gage, and were cotton covered. This small size of wire was used to minimize the effect of radiation to or from the couple, and the effect of the depth of its immersion. To prevent the serious effect of moisture or dew, between the cold storage room and the room in which the instruments were arranged the wires were all carefully insulated with rubber tape.

The junctions of the couples were all formed by fusing the two wires together in an oxyacetylene gas flame, making a button about the size of a pin head at the junction. A strong joint was made in this manner.

To guard against the effect of water in the concrete the wires leading to the hot junctions of the thermocouples were insulated with glass tubing and the joints themselves were put into melted paraffin so that the paraffin filled the tubing about two inches and covered the surface of the wires with a thin film at the joint.

At the cold junction the joint of the wires was put into a test tube filled with transformer oil, the wires being insulated from each other by glass tubing. A thermos bottle filled with ice shavings and water was used as the cold bath of the test tube.

The thermocouples were calibrated by means of a mercury thermometer which was graduated to one-tenth of one degree centigrade. The oil bath used for constant temperature in calibrating was a small test tube which was inserted into a thermos bottle. The calibration curves for all the couples were practically the same. Fig. 2 shows

the calibrating apparatus and a typical calibration curve. The discontinuity of the curve near the zero point is due to the instrumental error of the potentiometer used.

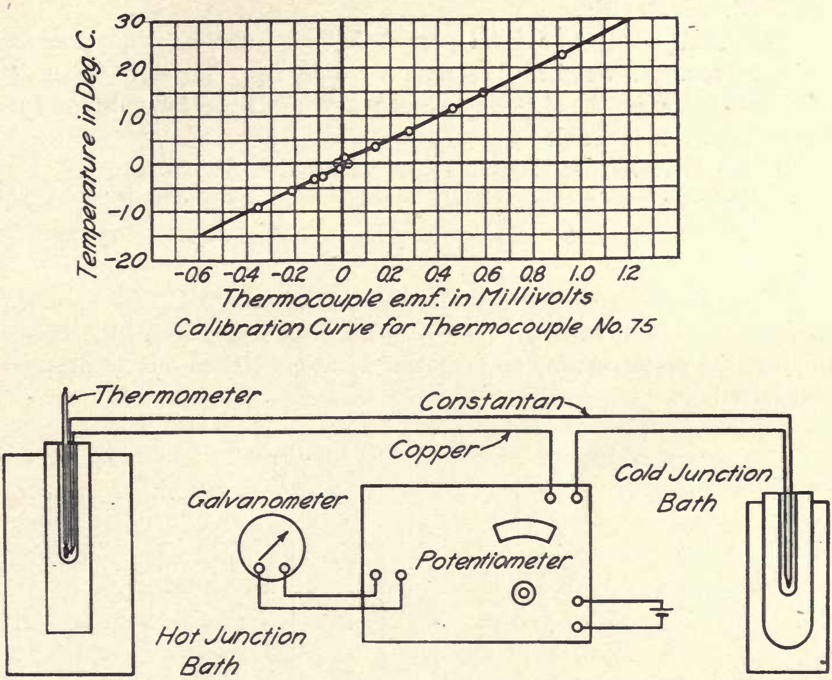

FIG. 2. CALIBRATING APPARATUS AND A TYPICAL CALIBRATION CURVE

The potentiometer gave a direct reading of 0.01 millivolt. To increase the accuracy of the balance of electromotive force, the galvanometer of the instrument was replaced by a D'Arsonval galvanometer.

As the precision of the thermocouple in use is about 25 deg. C. per millivolt, the precision attainable is 0.25 deg. C. It was found possible to repeat the readings to within 0.005 millivolt, or 0.13 deg. C. when the hot junction was placed in a constant temperature oil bath.

5. *Concrete Materials.*—The materials used were similar in character and quality to those used in other concrete and reinforced

concrete specimens made and tested by the Engineering Experiment Station in recent years.

Universal portland cement was used in all specimens. It fulfilled the requirements of the specification of the American Society for Testing Materials.

The sand used came from pits at Attica, Indiana, and passed a ¼-in. screen. It weighed 111 lb. per cubic ft. The sieve analysis of this sand, using the sieves commonly known as the Tyler Standard Sieves, is given in Table 1.

Clean gravel from the same pits was used. It passed a 1½-in. screen and was retained on a ¼-in. screen. The weight of this gravel was 99 lb. per cubic ft. A sieve analysis of the material is shown in Table 1.

Concrete of three proportions, 1-3-6, 1-2-4, and 1-1-2, by volume, was tested as indicated in Table 2. To insure accuracy in proportioning, the materials for each specimen were weighed out separately and mixed.

TABLE 1

SIEVE ANALYSIS OF SAND AND GRAVEL

(TYLER STANDARD SIEVES USED)

Material	Sieve Size	Size of Square Openings, inches	Percentage Passing*
Sand	⅜ in.	0.371	100
	No. 4	0.185	91
	" 8	0.093	66
	" 14	0.046	49
	" 28	0.0231	19
	" 48	0.0116	3
	" 100	0.0058	1
Gravel	1½ in.	1.49	100
	¾ "	0.742	57
	⅜ "	0.371	11
	No. 4	0.185	5
	" 8	0.093	3

* Each value represents the average of four tests.

The fineness modulus* and surface modulus* of these concrete aggregates are 5.98 and 4.69 respectively. From these figures it appears that the materials are rather coarse, but they are undoubtedly similar to the materials used in common practice. It is believed that the experimental data are applicable to any other well proportioned concrete.

* See Discussion on Proportioning of Concrete, by Professor D. A. Abrams and Professor A. N. Talbot. Proceedings of A. S. T. M., 1919. p. 477–485.

TABLE 2

PROPERTIES OF THE CONCRETE USED IN THE EXPERIMENTS

Proportion by Volume	Water-Cement Ratio	Slump Test on 6 by 12-in. Cylinders, inches	Twenty-eight Day Compression Tests of 6 by 12-in. Cylinders, lb. per sq. in.*
1-2-4 (all except Experiments Nos. 13, 14, 18, and 20)	1.01	2	1300
1-2-4 (Experiments Nos. 13 and 18) wet consistency.......	1.26	9	700
1-2-4 (Experiments Nos. 14 and 20) dry consistency......	0.81	0	2200
1-3-6 medium consistency....	1.29		800
1-1-2 medium consistency....	0.72	..	2900

* The cylinders were stored in damp sand at an average temperature of 70 deg. F. Each value is the mean of tests on three cylinders.

6. *Mixing of Concrete.*—The amount of water used in mixing the concrete was designed to produce the wettest concrete which could satisfactorily be used in freezing weather, except in Experiments Nos. 13 and 18, and 14 and 20, in which very wet and very dry mixtures, respectively, of 1-2-4 concrete were used. An idea of the mobility of these concretes may be obtained from the slump tests made on 6-in. by 12-in. cylinders noted in Table 2, as well as from the properties of the concrete aggregates and the water-cement ratio (ratio of volume of water to volume of cement) also shown in Table 2.

The mixing of the concrete was done by hand. The dry cement and sand were first mixed to a uniform color and spread out in a thin layer in a large mixing pan; the stone was then added, and the whole mass turned with shovels until thorough incorporation of the dry materials was secured; water was then added, and the materials turned until thoroughly mixed.

The temperature of the concrete thus made was practically uniform and was the same as the temperature of the room in which the concrete was mixed.

7. *General Features of the Experiments.*—Sixteen experiments were made to obtain data on the cooling of fresh concrete exposed to freezing temperatures. One experiment was made on concrete which was 31 days old.

Experiments Nos. 1-6, 10, and 13-15 were made with the purpose of determining the thermal constants of fresh concrete which are necessary in applying the theory of heat conduction to the cooling of fresh concrete, and others were made to obtain information on the effect of protection of the concrete surface. Experiment No. 17 was made to determine the thermal constants of concrete 31 days old. These experiments were made at the plant of the Smith Ice and Cold Storage Company at Champaign, Illinois.

The size of specimen, the concrete mixtures, and the conditions of the surface are given in Table 3.

TABLE 3
DATA OF THE SPECIMENS

Experiment No.	Proportion by Volume	Depth of the Specimen* cm.	Condition of the Surface
1	1-3-6	50	No protection at the surface under a still air condition.
2	1-2-4	50	"
3	1-1-2	50	"
4	1-3-6	30	"
5	1-2-4	30	"
6	1-1-2	30	"
7	1-2-4	30	Surface cooled by air current from a fan.
8	1-2-4	30	Surface covered with a board ¾-in thick.
9	1-2-4	30	Surface covered with canvas.
10	1-2-4	20	No protection at the surface.
11	1-2-4	20	Surface covered with a board.
12	1-2-4	20	Surface covered with canvas.
13	1-2-4	30	No protection at the surface.
14	1-2-4 (wet consistency)	30	"
15	1-2-4 (dry consistency)	25	"
16	1-2-4	10	Surface covered with a board.
17	1-2-4	30	No protection at the surface.

* All the specimens are 12-in. square in horizontal section.

8. *Preparation of Test Specimens.*—The mold for the test specimens was a wooden box 12 in. square and from 12 to 24 in. deep, in

inner dimensions. It was made of 1½-in. pine boards. This mold was placed inside another wooden box 3 ft. square and 3 ft. deep. The space between the two was filled with dry sawdust to prevent cooling of the fresh concrete at the sides and at the bottom of the mold.

The inner surface of the inner box was coated with Parowax to prevent the absorption of water from the concrete by the wood.

As each specimen was being poured, the thermocouples were inserted one by one through small holes in the side of the mold and embedded in the concrete, their joints being at specified depths below the upper surface and in the vertical axis of the specimen. A thermocouple was also embedded in the sawdust to measure its cooling. The temperatures of the concrete, the boxes, and the sawdust were practically the same as the temperature of the room at the beginning of all experiments.

The specimen thus made was removed to the adjoining cold storage room about twenty minutes after the water was added to the concrete mixture. The wires of the thermocouples were carried across the door sill from the cold room to the room in which the measuring instruments were arranged. The temperature of the cold room was measured by a thermocouple. Fig. 1 shows in diagrammatic form the general arrangement of the apparatus.

9. *Phenomena of Experiments.*—In experiments Nos. 1, 2, and 3, specimens 50 cm. in depth, of 1-3-6, 1-2-4, and 1-1-2 concrete, respectively, were tested. From the data of these experiments Fig. 3 has been plotted. It gives the curves of cooling at the several depths below the exposed surface of the specimen. The abscissas give the elapsed time from the date the specimen was placed in the cold room. The temperature of the cold room is also shown in the diagram. The effect of the heat produced during the setting of the concrete is noticeable in the figure even in the case of the 1-3-6 mixture.

In these first three experiments the thermocouple measuring the temperature of the sawdust at the underside of the mold always showed a lower temperature than that at the bottom of the specimens, and it was clear that there was a flow of heat through the bottom of the mold from the specimen to the sawdust. The rise in temperature during the setting of the concrete was also marked with so great a depth of specimen. For these reasons, the data obtained from these three experiments were not used in calculating the thermal constants.

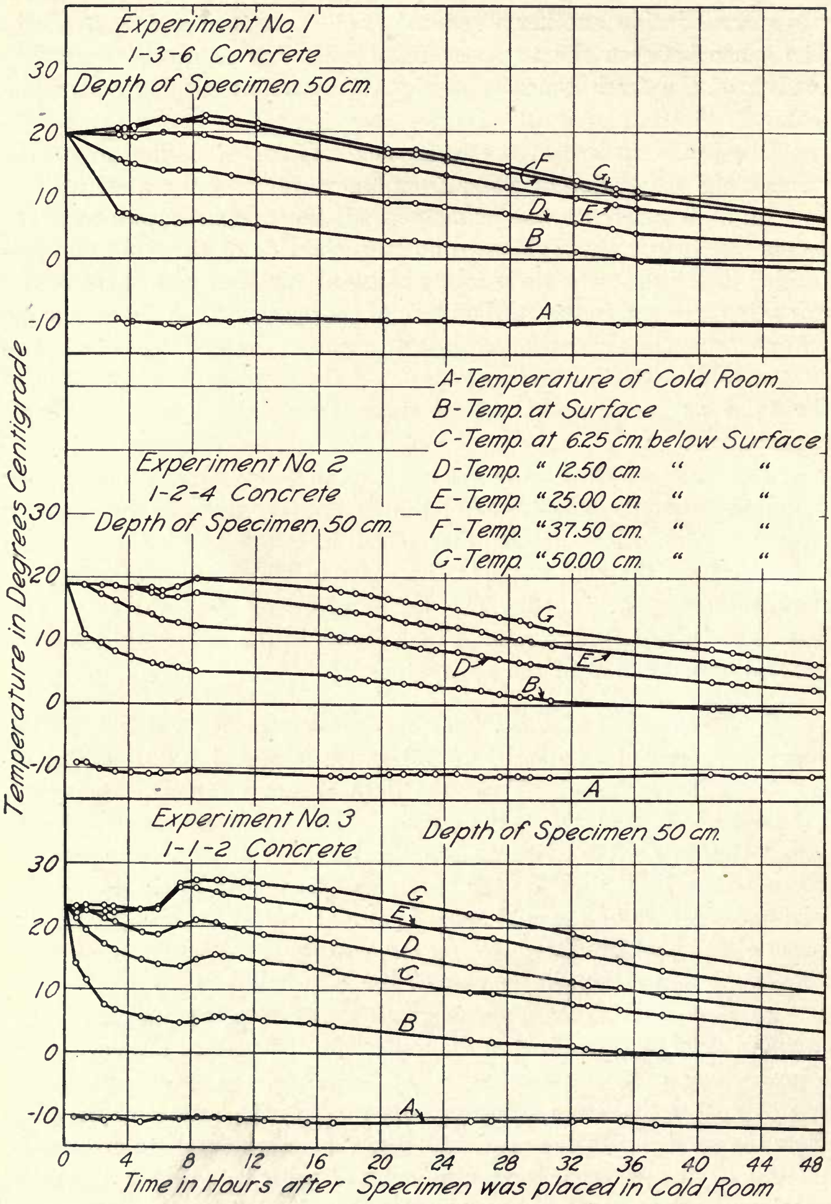

Fig. 3. Curves of Cooling, Experiments 1, 2, and 3

In experiments Nos. 4, 5, and 6, specimens 30 cm. thick and of 1-3-6, 1-2-4, and 1-1-2 concrete, respectively, were used. This depth was used in order to reduce the effect of the rise in temperature produced in the setting of the concrete, and also to minimize the flow of heat through the bottom of the mold. For the latter reason further insulation was obtained by filling in the bottom of the mold with a 3-in. layer of sawdust, placing upon this a ¾-in. pine board, and adding a 1½-in. layer of Parowax. The inner sides of the mold were also given a covering of Parowax ⅛ in. thick. With these precautions the thermocouple in the sawdust below the bottom of the mold showed practically the same temperature as that of the bottom of the specimen, and it was considered that the bottom of the mold was thus kept sufficiently impervious to the flow of heat. This condition was assumed in calculating the thermal constants of the concrete from the test data. The curves of cooling for these three specimens are plotted in Fig. 4.

The specimens used in experiments Nos. 7, 8, and 9 were of the same material as that used in experiment No. 5; they differed in surface conditions.

In experiment No. 7 the surface of the specimen was subjected to an air current produced by an ordinary electric fan operating at its medium speed. The distance between the fan and the concrete was about one foot, the direction of the current making an angle of about 30 degrees with the surface. The velocity of the air current was about ten miles per hour. It was intended by this method to obtain information on the effect of wind on the cooling of fresh concrete. The cooling of the concrete surface was very striking. The time required for the freezing temperature to penetrate the surface in this experiment was about one-tenth of that under a still air condition. It is clear that the evaporation of water from the surface had a very great effect on the cooling of the surface; within the mass the effect was slight. Fig. 5 shows the results of the experiment.

In experiment No. 8 the surface of the specimen was covered with a pine board 1.9 cm. (¾ in.) thick. The small opening between the edge of the board and the box was sealed with Parowax. This experiment was made to determine the effect of wood forms on the cooling of fresh concrete. The board protected the surface very well. The results are plotted in Fig. 5.

In experiment No. 9 the top of the inner and outer boxes was covered with 10-oz. duck. There was about a 3-in. air space between

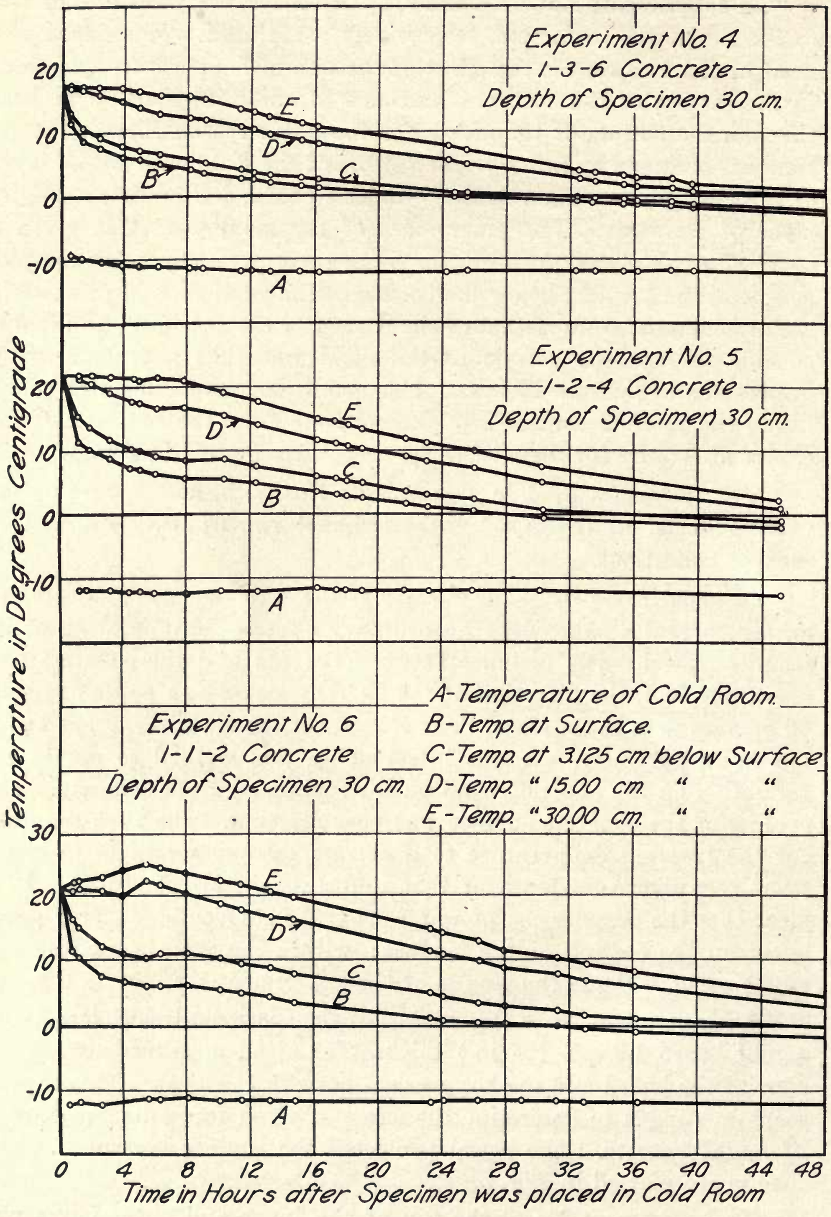

FIG. 4. CURVES OF COOLING, EXPERIMENTS 4, 5, AND 6

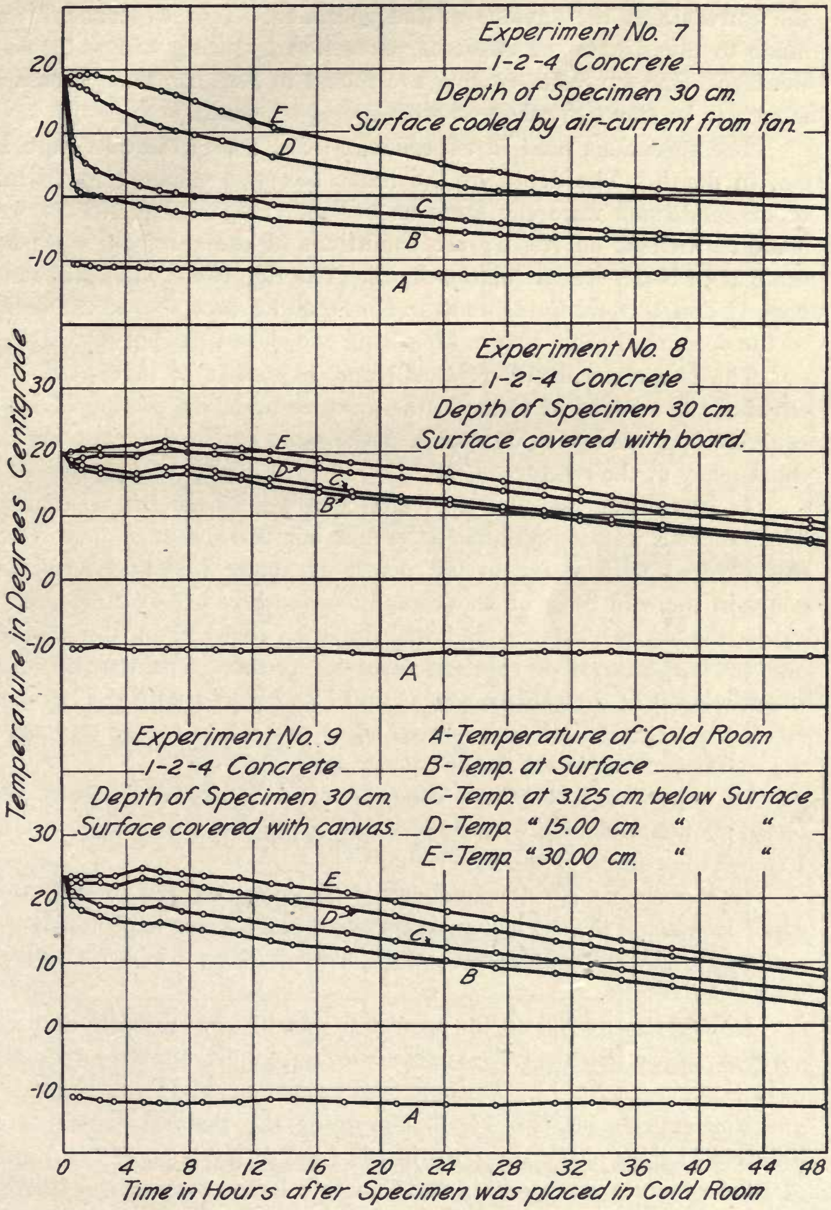

FIG. 5. CURVES OF COOLING, EXPERIMENTS 7, 8, AND 9

the canvas and the surface of the specimen. This experiment was made to investigate the effect of protection by using canvas in cold weather. The curves of cooling are shown in Fig. 5. The protection is seen to be about the same as with the ¾-in. board.

The specimens used in experiments Nos. 10, 11, and 12 were 20 cm. in depth. The depth of the inner box was changed to 16 in.; other conditions were the same as before. Experiment No. 10 was made as a check on No. 5; the conditions of the specimen were the same as in No. 5 except for the depth. The conditions in experiments Nos. 11 and 12 were the same as in Nos. 8 and 9 except as to the depth of the specimen. The curves of cooling are shown in Fig. 6.

The two experiments (Nos. 13 and 14) made to investigate the effect of the amount of water in the mixture upon the cooling of fresh concrete had conditions of specimen the same as No. 4 except for the consistency of the mixture.

In experiment No. 13, 1-2-4 concrete of wet consistency was tested. The concrete was so wet that after making the specimen its surface was covered with water to the depth of about 3/16-in. After six hours in the cold room at an average temperature of –12.2 deg. centigrade, the surface was covered with laitance about ⅜-in. thick which had the appearance of soft chocolate ice cream. The thermocouple which was set at the surface was at first also covered with the laitance. It may be that the laitance offered some protection to the cooling of the surface. The curves of cooling are shown in Fig. 7.

In experiment No. 14 a concrete of very dry consistency was tested; much difficulty was experienced in placing this concrete. Fig. 7 shows the results of the experiment.

In experiment No. 15 the depth of the specimen was 25 cm., other conditions being the same as in experiment No. 5; this experiment was made as a check on the results of experiments Nos. 5 and 10. Fig. 7 shows the results.

In experiment No. 16 the specimen was 10 cm. in depth, and was covered with a board at the surface; other conditions were the same as in experiment No. 11. The curves of cooling are shown in Fig. 8.

For experiment No. 17, to determine the thermal constants of concrete 31 days old, a specimen in the form of a 30-cm. (12-in.) cube of 1-2-4 concrete was made in the Concrete Laboratory at the University. Two thermocouples were used at both the surface and the bottom and one at the center. Two days after the specimen was made

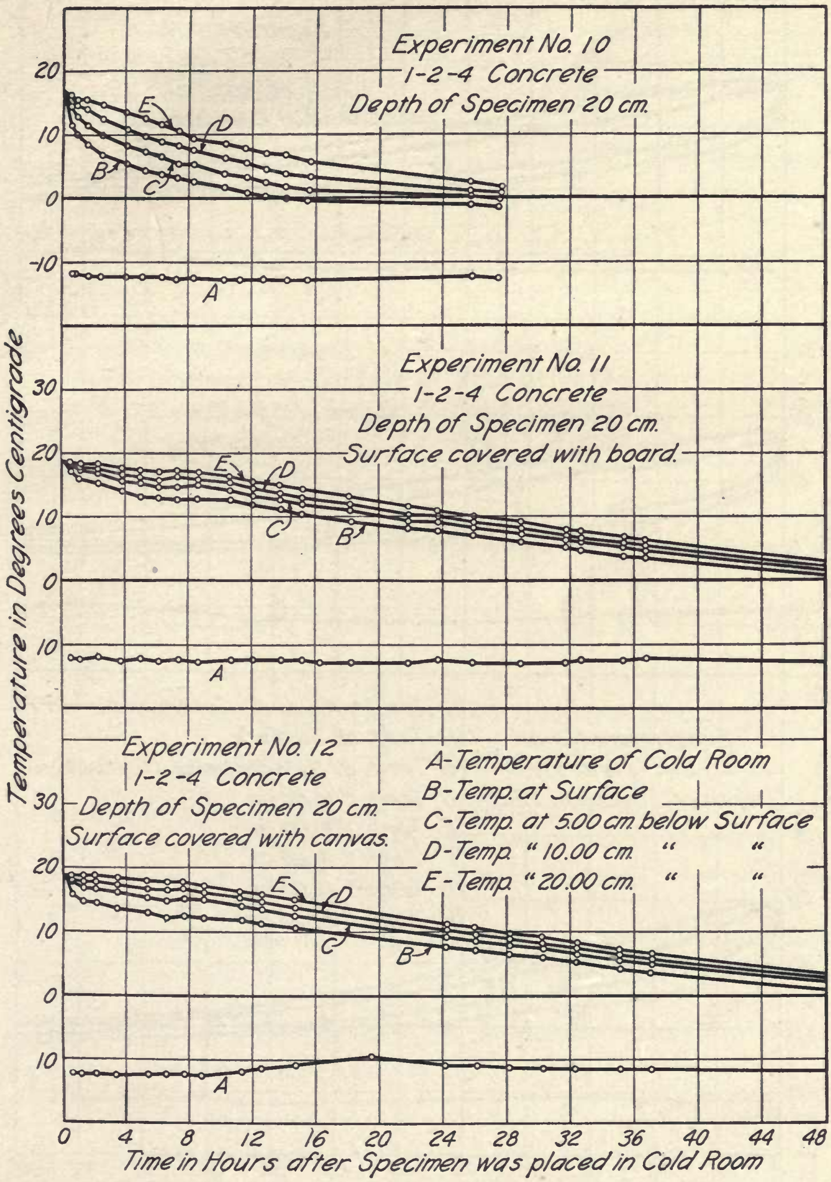

FIG. 6. CURVES OF COOLING, EXPERIMENTS 10, 11, AND 12

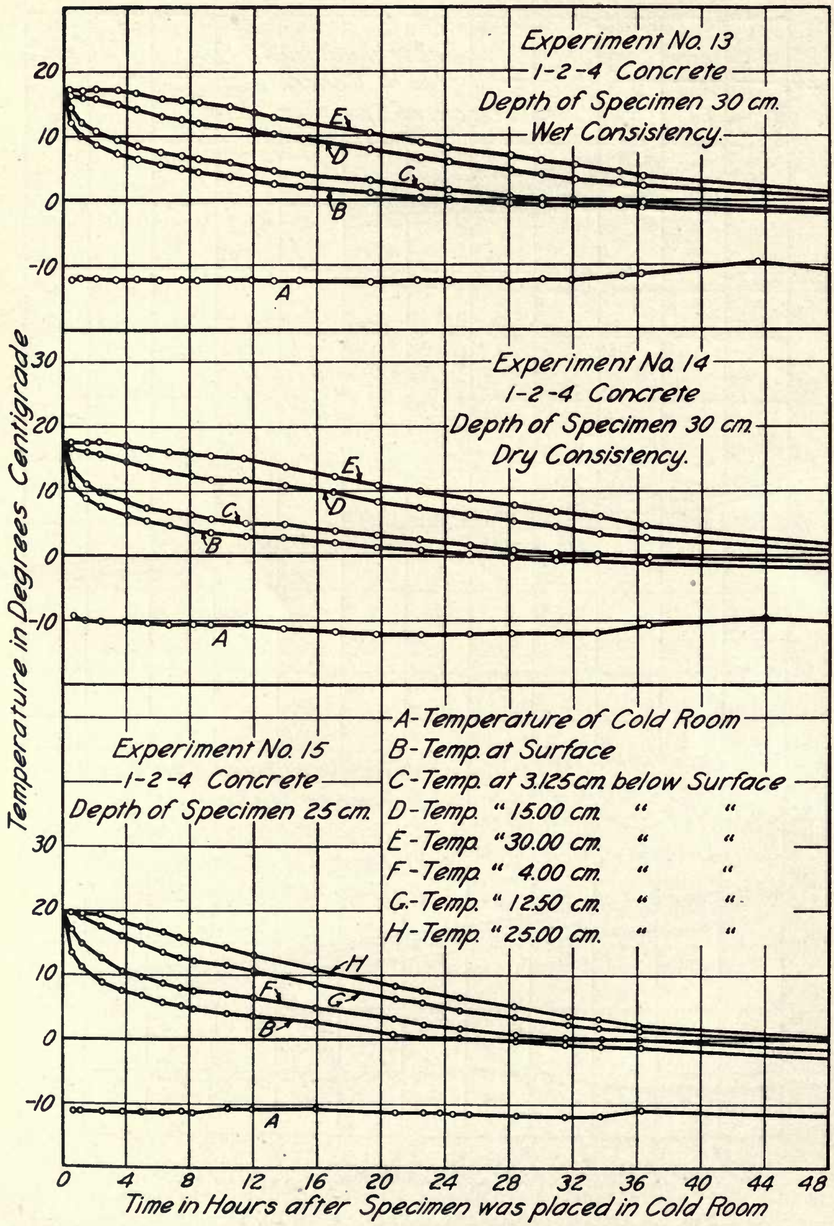

FIG. 7. CURVES OF COOLING, EXPERIMENTS 13, 14, AND 15

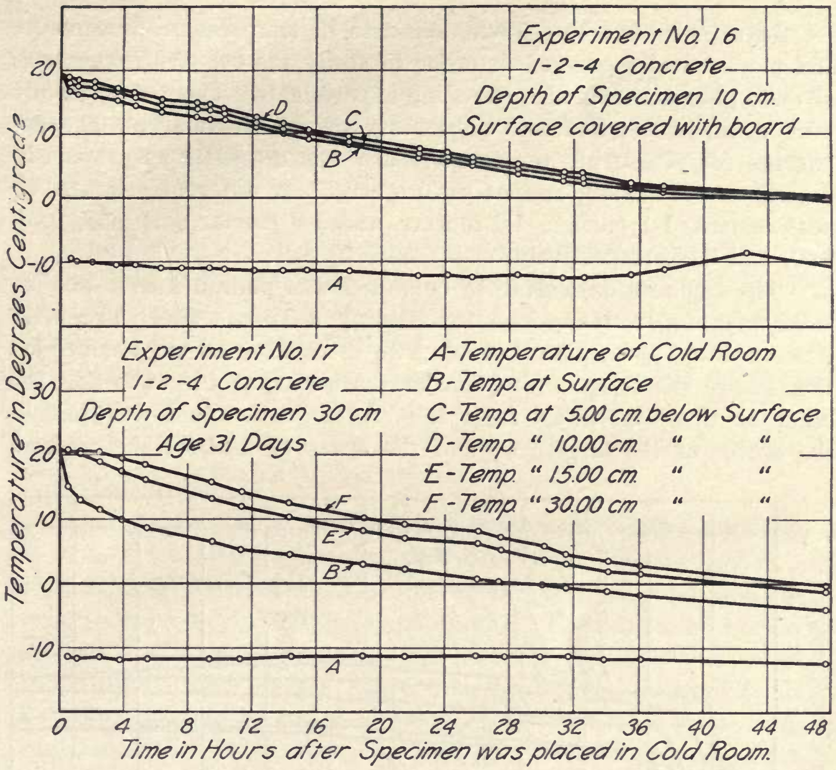

FIG. 8. CURVES OF COOLING, EXPERIMENTS 16 AND 17.

the forms were taken off, and the cube was stored in damp sand. After 24 days it was taken out of the sand and dried in the laboratory for one week. It was then put in the wooden mold, the small spaces between the specimen and walls of the mold being filled with Parowax.

The specimen and the temperature measuring apparatus were arranged as in experiment No. 1. Before it was carried into the cold room the temperature of the specimen at different points was about the same, and the mean was taken as the initial temperature of the specimen. The curves of cooling are shown in Fig. 8.

10. *Rise in Temperature During Setting of Concrete and Mortar.* —The chemical combination of water with portland cement is an exothermic reaction, the heat evolved being sufficient to raise materially the temperature of concrete and mortar during the period of setting and hardening.

In experiments Nos. 18-26, this rise in temperature of concrete and mortar was measured in order to study the relative importance of these phenomena in the preceding experiments. These experiments were made in the Concrete Laboratory of the University. The apparatus for measuring the temperature was the same as previously described. The water-cement ratio of the four different mixtures of neat cement, 1-1 mortar, 1-2 mortar, and 1-3 mortar was 0.38, 0.48, 0.60, and 0.85, respectively.

The concrete or mortar to be tested was poured into a wooden mold with inner dimensions for holding a 12-in. cube. The mold was made of 1½-in. pine boards. Two thermocouples were embedded to measure the temperature, one at the center of the cube and the other at three in. away from the center. The mold was put in the center of the large box, and the space around all six surfaces

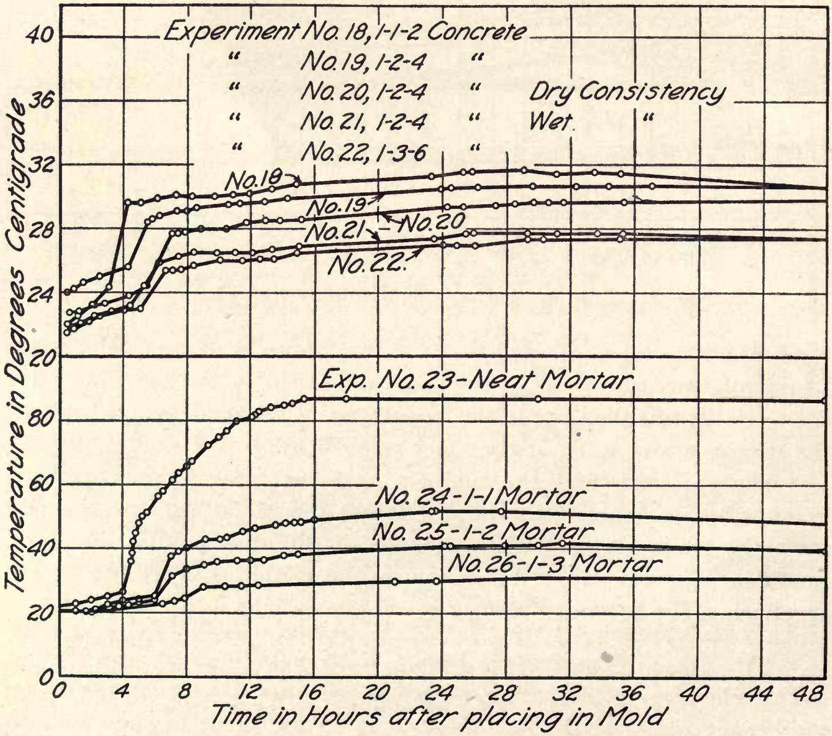

FIG. 9. DIAGRAMS SHOWING RISE IN TEMPERATURE DURING SETTING OF CONCRETE AND MORTAR

of the mold was filled with sawdust about 8 in. thick, to prevent the cooling of the specimen.

The initial temperature of the fresh concrete or mortar was the same as that of the laboratory. The two thermocouples showed about the same temperature, and the temperature at the center is plotted in Fig. 9. After 48 hours the rise in temperature was very small in all cases.

The total rise in temperature, the rate of increase, and the time interval before the maximum temperature was reached were all variable, depending upon the proportions of the mixture, the amount of water used in the mixing, and the initial temperature of the concrete or mortar. The curves in the figure, however, show very clearly that considerable heat is produced during the setting of concrete or mortar at ordinary temperatures, and its effect is very noticeable from 6 to 12 hours after the pouring of the concrete.

11. *Calculation of the Thermal Constants of Fresh Concrete.*—In order to apply the theory of heat conduction to concreting in cold weather, it is necessary to determine certain thermal properties of the concrete from the preceding experiments. Even in a homogeneous material such thermal properties are variable and may be considered constant only for a certain range in temperature; in fresh concrete the properties will be found to vary with other conditions as well as temperature. It is the purpose here to determine certain thermal properties for a limited range of temperatures near freezing, and for purposes of application these properties will be termed "thermal constants."

From the time that water is added to the concrete materials, a chemical reaction begins with an accompanying rise in temperature, and the mass solidifies after a few hours under ordinary temperatures. In temperatures below about 10 deg. C. (50 deg. F.) the setting takes place very slowly and produces a small amount of heat, while if the temperature falls below 0 deg. C. (32 deg. F.) the fresh concrete will freeze and little setting can be expected. Therefore, even though the effect of heat produced in the setting of concrete be neglected, the coefficient of thermal conductivity of fresh concrete will vary from time to time according to the degree of its setting and its temperature. The specific heat and density of the material also vary considerably with different conditions of setting, as well as with the mix of the concrete and the temperature.

When the surface of the concrete is worked and finished, there is more or less water remaining on it. Therefore, the cooling at the surface of concrete is due not only to radiation and convection, but also to evaporation of water, which, of course, depends upon the humidity and other conditions of the surrounding air.

It is obviously impossible to give certain and definite values of the thermal constants of fresh concrete. However, it is considered useful to determine fair average values of the constants which may be used for practical problems. The execution of concrete work in general practice does not compare in refinement with the measurement of temperature to one degree centigrade or of a time interval to one minute. It is usually possible, however, to make observations with an accuracy of within a few degrees in temperature or a few hours in time, and this usually will be sufficient for the application of information regarding concreting in freezing weather. Hence, the constants determined here should be useful in applying the mathematical theory of heat conduction to practical problems.

The method used in calculating the thermal constant involves three things: the use of well established fundamental formulas applying to various materials, a rigorous definition of the assumptions which may be made regarding the conditions in these experiments, and a reduction of the fundamental equations to special forms which satisfy the assumptions and provide the desired working formulas.

In deriving the fundamental formulas, a body will be considered which has infinite length and breadth and a finite thickness. For convenience in calculation the two surfaces are considered to be exposed to air at zero temperature, although the results of the analysis may be readily applied with any other air temperatures. The C. G. S. system of units is used in the following treatment.

Let $v =$ the temperature at any point in the body at a distance x from one of the surfaces at any time, t,

$v_0 =$ the initial temperature of the body, that is, the temperature at all points at the time $t = 0$,

$k =$ the diffusivity* of the body,

$$= \frac{\text{(coefficient of thermal conductivity)}}{\text{(density)}\ \ \text{(specific heat)}}$$

$$h = \frac{\text{(emissivity)*}}{\text{(coefficient of thermal conductivity)}}$$

* For definitions and a more detailed treatment of the analysis, see Carslaw, "Fourier's eries and Integrals." 1906.

where emissivity of the body is the rate of loss of heat by radiation and convection, under certain conditions of air, per square centimeter of surface, per degree difference in temperature between body and air, assuming Newton's law of cooling.

Then the equation of temperature in the body is

$$\frac{\partial v}{\partial t} = k\frac{\partial^2 v}{\partial x^2} \qquad 0 < x < l$$

As the initial and boundary conditions we have,

$$v = v_o \qquad \text{for } t = 0$$

$$-\frac{\partial v}{\partial x} + h\,v = 0 \qquad \text{at} \qquad x = 0,$$

$$\frac{\partial v}{\partial x} + h\,v = 0 \qquad \text{at} \qquad x = l,$$

The solution* of these equations is

$$v = 2v_o\sum_{n=1}^{n=\infty} e^{-k\alpha_n^2 t}\frac{\alpha_n \cos\alpha_n x + h\sin\alpha_n x}{(\alpha_n^2 + h^2)l + 2h}\int_o^l (\alpha_n\cos\alpha_n x + h\sin\alpha_n x)\,dx. \tag{1}$$

in which α_n is the n th positive root of the equation

$$\tan\alpha l = \frac{2\alpha h}{\alpha^2 - h^2}$$

excluding the root $\alpha = 0$.

At $x = 0$, we have

$$v_s = 2v_o\left\{e^{-k\alpha_1^2 t}\frac{\alpha_1}{(\alpha_1^2 + h^2)l + 2h}\int_o^l (\alpha_1\cos\alpha_1 x + h\sin\alpha_1 x)\,dx + \right.$$

$$\left. e^{-k\alpha_2^2 t}\frac{\alpha^2}{(\alpha_2^2 + h^2)l + 2h}\int_o^l (\alpha_2\cos\alpha_2 x + h\sin\alpha_2 x)\,dx + \ldots\ldots\right\}$$

At $x = \dfrac{l}{2}$, since

* Carslaw, "Fourier's Series and Integrals," page 274, 1906.

$$\tan \alpha_n l = \frac{2\,\alpha_n h}{\alpha_n^2 - h^2}$$

$$\tan \frac{\alpha_n l}{2} = + \frac{h}{\alpha_n} \quad \text{when } n \text{ is odd,}$$

$$\tan \frac{\alpha_n l}{2} = - \frac{\alpha_n}{h} \quad \text{when } n \text{ is even.}$$

Thus, for $x = \frac{l}{2}$

$$(\alpha_n \cos \alpha_n x + h \sin \alpha_n x) = \alpha_n \left\{ \cos \frac{\alpha_n l}{2} + \frac{h}{\alpha_n} \sin \frac{\alpha_n l}{2} \right\}$$

$$= \alpha_n \cos \frac{\alpha_n l}{2} \left(1 + \frac{h}{\alpha_n} \tan \frac{\alpha_n l}{2} \right)$$

$$= \frac{\alpha_n}{\cos \frac{\alpha_n l}{2}} \quad \text{when } n \text{ is odd, and}$$

$$= 0 \quad \text{when } n \text{ is even.}$$

Thus

$$v_{\frac{l}{2}} = 2\,v_o \left\{ e^{-k\,\alpha_1^2 t} \frac{\frac{\alpha_1}{\cos \frac{\alpha_1 l}{2}}}{(\alpha_1^2 + h^2) l + 2h} \int_o^l (\alpha_1 \cos \alpha_1 x + h \sin \alpha_1 x) dx + \cdots \right\}$$

As $k\,\alpha_n^2 t$ increases with n, if t is chosen great enough, we shall obtain a close approximation by using only the first term in each of these series.

Thus

$$\frac{v_s}{v_{\frac{l}{2}}} = \cos \frac{\alpha_1 l}{2} \quad \ldots\ldots (2)$$

From this result we find α_1 ; and h follows from the equation

$$\tan \frac{\alpha_1 l}{2} = \frac{h}{\alpha_1} \quad \ldots\ldots (3)$$

And k may be found from either of the equations

$$e^{-k\alpha_1^2 t} = \frac{v_s}{2v_o} \frac{(\alpha_1^2 + h^2)l + 2h}{\alpha_1 \left\{ \sin\alpha_1 l + \frac{h}{\alpha_1}(1 - \cos\alpha_1 l) \right\}} \qquad (4)$$

$$e^{-k\alpha_1^2 t} = \frac{v_{\frac{l}{2}}}{2v_o} \frac{\left\{ (\alpha_1^2 + h^2)l + 2h \right\} \cos\frac{\alpha_1 l}{2}}{\alpha_1 \left\{ \sin\alpha_1 l + \frac{h}{\alpha_1}(1 - \cos\alpha_1 l) \right\}} \qquad (5)$$

Thus, if the values of v_s and $v_{\frac{l}{2}}$ are measured at a certain time t, the thermal constants k and h can be determined by equations (2) to (5).

To apply the experimental data to equations (2) to (5), the following assumptions have been made:

(1) The diffusivity of fresh concrete is assumed a constant.

(2) The emissivity of fresh concrete is defined as a constant rate of loss of heat by radiation, convection, and evaporation of water on the surface (that is, the effect of all agents which have relation to the conditions of the surface) per square centimeter of surface under still air conditions per degree difference in temperature between body and air.

(3) The bottom of the specimen is assumed to be impervious to heat, i. e. there is no transference of heat across the bottom plane of the specimen.

(4) The cooling in the vertical axis of the specimen is assumed to be due entirely to the cooling at the surface of the specimen.

(5) The initial temperature of the concrete, i. e. the temperature of the concrete just before the specimen was put into the cold room, is assumed constant throughout the specimen.

(6) For convenience of calculation, the mean temperature of the cold room until the time considered in the calculation is taken as the constant temperature of the cold room.

(7) The heat produced during setting clearly retards the cooling of fresh concrete. However, with a comparatively small specimen, in a cold temperature, this effect is not so noticeable.

It is assumed that this effect is included in the constant, diffusivity, of the concrete.

Now let

$-v_2 =$ the mean temperature of the cold room until the time t.

$v_1 =$ the initial temperature of the concrete.

Then the difference between the initial temperature of the concrete and the mean temperature of the cold room until the time considered is $v_0 = v_1 + v_2$.

As the cooling of the body is dependent upon the difference of temperature, it may be considered that the concrete of temperature v_0 cools under the air at 0 deg. C.

It has been assumed that there is no flow of heat across the bottom plane of the specimen. If we imagine a wall having the thickness equal to twice the depth of the specimen and the temperature of air on both sides of the wall is zero, then there will be no transference of heat across its middle plane because of the symmetrical condition of the wall, and hence this plane could be made of a material impervious to heat without altering the conditions. Therefore, we may consider that the specimen is one-half of that wall, and if we take the thickness of the wall as twice the depth of the specimen, equations (2) to (5) apply to the specimens of the experiments until the freezing temperature penetrates the surface. Hence the thermal constants k and h may be calculated by these equations.

When the freezing of fresh concrete begins latent heat must be taken into account. The problem is very complicated and it is not considered here.

The results of the calculation of the thermal constants k and h are given in Table 4.

For 1-2-4 concrete the mean values of k and h, calculated for the time when the freezing temperature penetrated the surface in experiments Nos. 5, 10, and 15 are 0.00626 and 0.0436, and the means of the values calculated at different times in the three experiments are 0.00628 and 0.0457 respectively.

For 1-3-6 concrete, the values of k and h calculated for the time when the temperature of the surface of the specimen reached zero are 0.00626 and 0.0428, and the means of the values calculated at different times are 0.00641 and 0.0466 respectively.

As the depth of the specimens used in the calculation is not large and the effect of heat produced during the setting of concrete is

not noticeable, the values 0.0063 and 0.046 seem to be safe average values of k and h respectively, for concrete of 1-2-4 and 1-3-6 mixtures having a consistency that can satisfactorily be used in cold weather. For massive concrete work the heat evolved during the setting of the concrete will retard the cooling considerably, and the values obtained will err on the side of safety.

For 1-1-2 concrete, as the heat produced during the setting of the concrete had a great effect, it was considered absurd to apply equations (2) to (5) for calculation of the thermal constants; but, of course, it is very safe to use the above values for construction in which such a rich mixture is used.

The 1-2-4 concrete of wet consistency shows lower values of the thermal constants than that of medium consistency. This is probably due to the protecting effect of the laitance on the surface in that experiment, and also to the high specific heat of water. It is considered that the values for the concrete of medium consistency will apply very safely to cases of wet consistencies until the freezing of the concrete takes place.

For the concrete of dry consistency the diffusivity is lower than that for concrete of other consistencies. The slightly higher value of h is considered to be due to the lower value of the coefficient of conductivity of this concrete.

For the concrete 31 days old the mean values of the constants k and h, calculated at different times, are 0.0083 and 0.032 respectively. In this experiment the cooling of the specimen through the sawdust was apparent, and the calculated values are considered to be higher than they should be.

12. *Effect of Wood Forms in Protecting Fresh Concrete.*—The wood used in forms is a poor conductor of heat and protects fresh concrete from cooling in cold weather. In addition to this, so-called contact or surface resistance, which is offered to the flow of heat by the discontinuity of the concrete at the forms, also serves to retard the cooling of the fresh concrete.

The coefficient of thermal conductivity of pine wood which is largely used in form work may be taken as about 0.00009 across the grain and 0.0003 with the grain.* In a board the flow of heat is not only across the fibre, but also along annular rings, depending upon

* Ingersoll and Zoebel, "Mathematical Theory of Heat Conduction," page 162, 1906.

TABLE 4

VALUES OF THERMAL CONSTANTS *h* AND *k* FOR FRESH CONCRETE

Experiment No.	Mixture	Depth of Specimen cm.	Time after Placing in Cold Storage hr.	min.	*h*	*k*
5	1-2-4	30	12	23	0.0454	0.00611
			23	4	0.0464	0.00653
			*28	18	0.0444	0.00624
				m	ean 0.0454	0.00628
10	1-2-4	20	10	20	0.0464	0.00684
			11	42	0.0472	0.00653
			*14	10	0.0470	0.00630
				m	ean 0.0467	0.00656
15	1-2-4	25	12	00	0.0479	0.00620
			20	46	0.0465	0.00596
			24	54	0.0467	0.00559
			*26	00	0.0393	0.00625
				m	ean 0.0451	0.00600
13	1-2-4 (wet consistency)	30	11	58	0.0429	0.00626
			19	20	0.0420	0.00586
			*24	15	0.0403	0.00570
				m	ean 0.0417	0.00594
14	1-2-4 (dry consistency)	30	11	30	0.0542	0.00513
			19	46	0.0483	0.00532
			*25	32	0.0466	0.00524
				m	ean 0.0497	0.00523
4	1-3-6	30	12	7	0.0480	0.00668
			12	12	0.0490	0.00628
			*24	00	0.0428	0.00626
				m	ean 0.0466	0.00641

* The temperature at the surface became zero at this time.

how the board was cut from the log. It is therefore obvious that the diffusivity of a board is quite variable and that the effect of a board in preventing the cooling of fresh concrete will vary considerably.

The emissivity of the same wood is about 0.0002 under still air conditions and about four times this value when exposed to a wind having a velocity of fifteen miles per hour.*

For these reasons, the more complicated mathematical solution of this case was not developed. However, in order to obtain some idea

*Harding and Willard, "Mechanical Equipment of Buildings," page 645.

of the effect of wood forms on the cooling of fresh concrete, the thickness of the board was assumed as equivalent in effect to a certain thickness of concrete.

Each specimen in experiments Nos. 8, 11, and 16 may be considered as one-half of a wall having a thickness of twice the depth of the specimen and having both faces of the wall protected by boards 1.9 cm. (3/4 in.) in thickness. Curves showing temperatures at different depths from the surface of the walls for various intervals of time, taking the initial temperature difference of air and concrete as unity, are plotted in Fig. 10.

Assuming a value of h, curves of temperatures corresponding to the above figures were calculated from equation (1) and plotted for different thicknesses of wall using the value of diffusivity of concrete as $k = 0.0063$. These curves and the curves of Fig. 10 were compared to find out at what values of h and thickness of wall the former approximated the latter. By this tentative method it was found that when a thickness of 11.25 cm. for the concrete was used in place of 1.9 cm. for the board and h equal to 0.031, the curves coincided very closely for the 48-hour period, but not so well for the 12-hour period.

On the whole, considering the entire range of the experiments, a thickness of concrete five times the thickness of a wood board may be considered equivalent in effect to the board, using $h = 0.031$ for the latter.

13. *Effect of Canvas in Protecting Fresh Concrete.*—The mathematical explanation of this effect is more difficult than the above case, and from the results of experiments Nos. 9 and 12 no general conclusion could be made. However, the curves shown in Fig. 11, which are based on the results of the experiments, are believed to give some information on the protection afforded by the use of canvas.

14. *Effect of Amount of Water Used.*—The curves of temperature in Fig. 12 which are based on the results of experiments Nos. 14, 5, and 13 for very dry, medium, and very wet consistencies of concrete, respectively, show that the more water used in mixing the concrete the slower the cooling is until the freezing of the fresh concrete begins. However, when the strength of concrete and the condition of the concrete after freezing are taken into account, it will be far better and safer in mixing concrete to use the least amount of water that will produce a workable mixture.

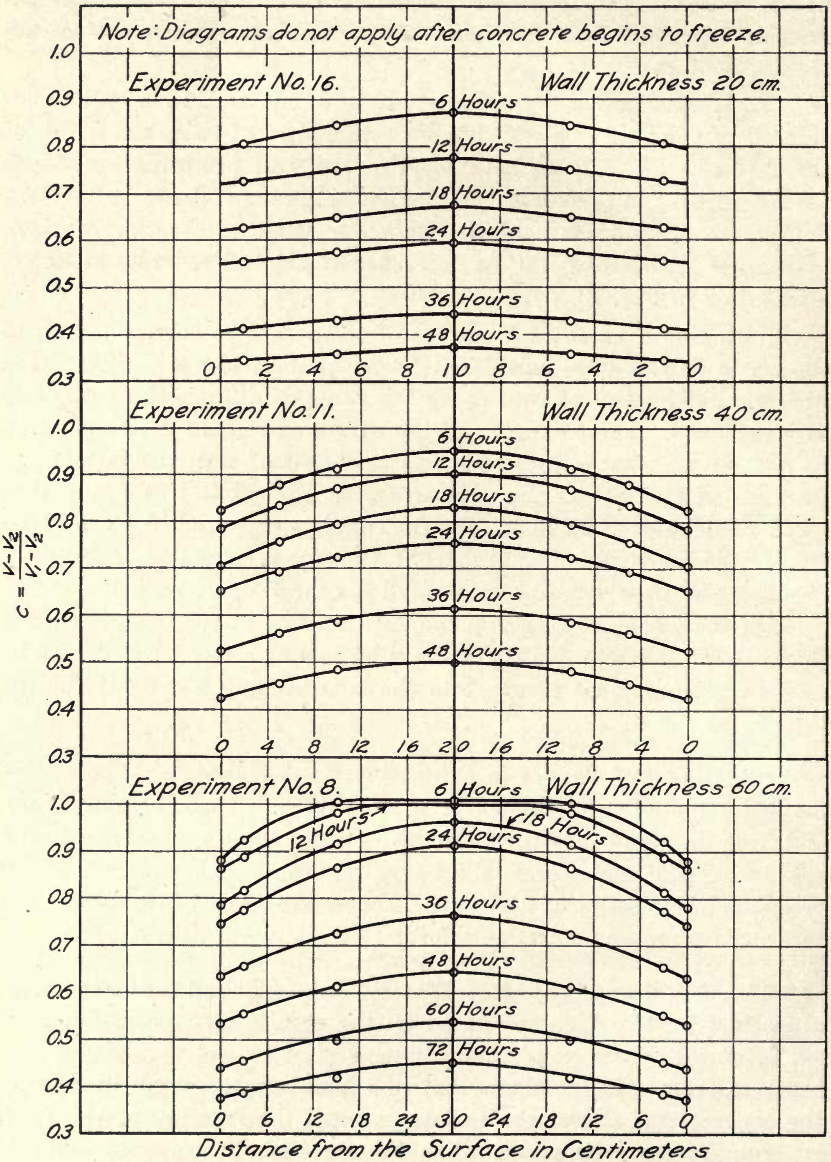

FIG. 10. DIAGRAMS FOR COOLING OF WALLS PROTECTED BY ¾-IN. WOOD FORMS

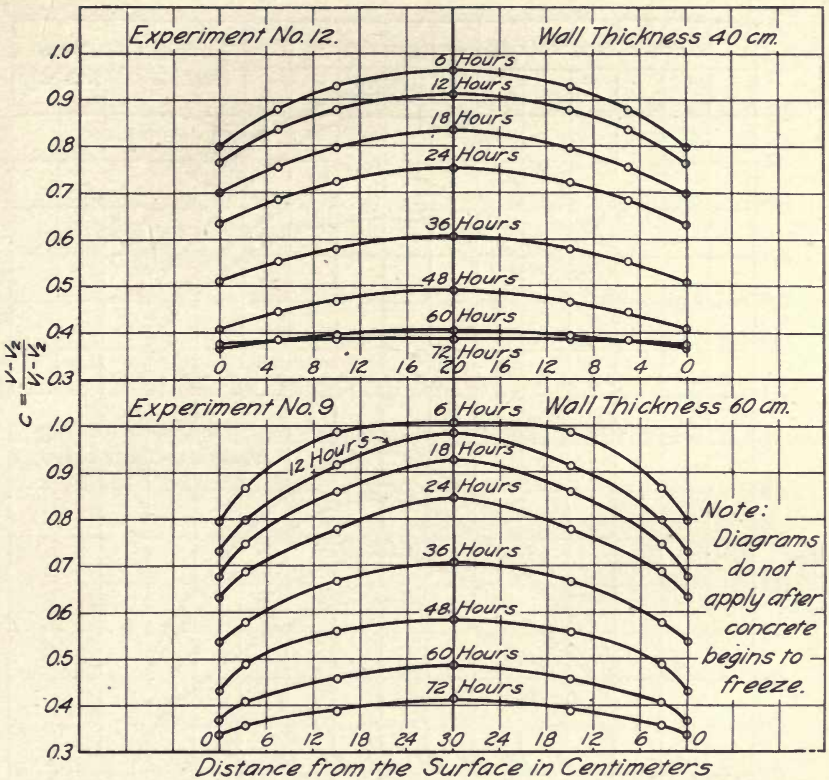

FIG. 11. DIAGRAMS FOR COOLING OF WALLS PROTECTED BY CANVAS

15. *Summary of Experimental Results.*—It is believed that the following general conclusions are instructive:

(1) For the application of the mathematical theory of heat conduction to the cooling of fresh concrete, the thermal constants, diffusivity and the ratio of the emissivity to the coefficient of conductivity, for commonly used concrete mixtures having the wettest consistency which can satisfactorily be used in cold weather and a still air condition, may be taken, in C. G. S. units, as 0.0063 and 0.046 respectively. These figures are especially safe for massive concrete work and for rich concretes.

(2) Under favorable temperature conditions in the concrete, the rise in temperature during its setting is greatest in the period between six and twelve hours after the time of mixing.

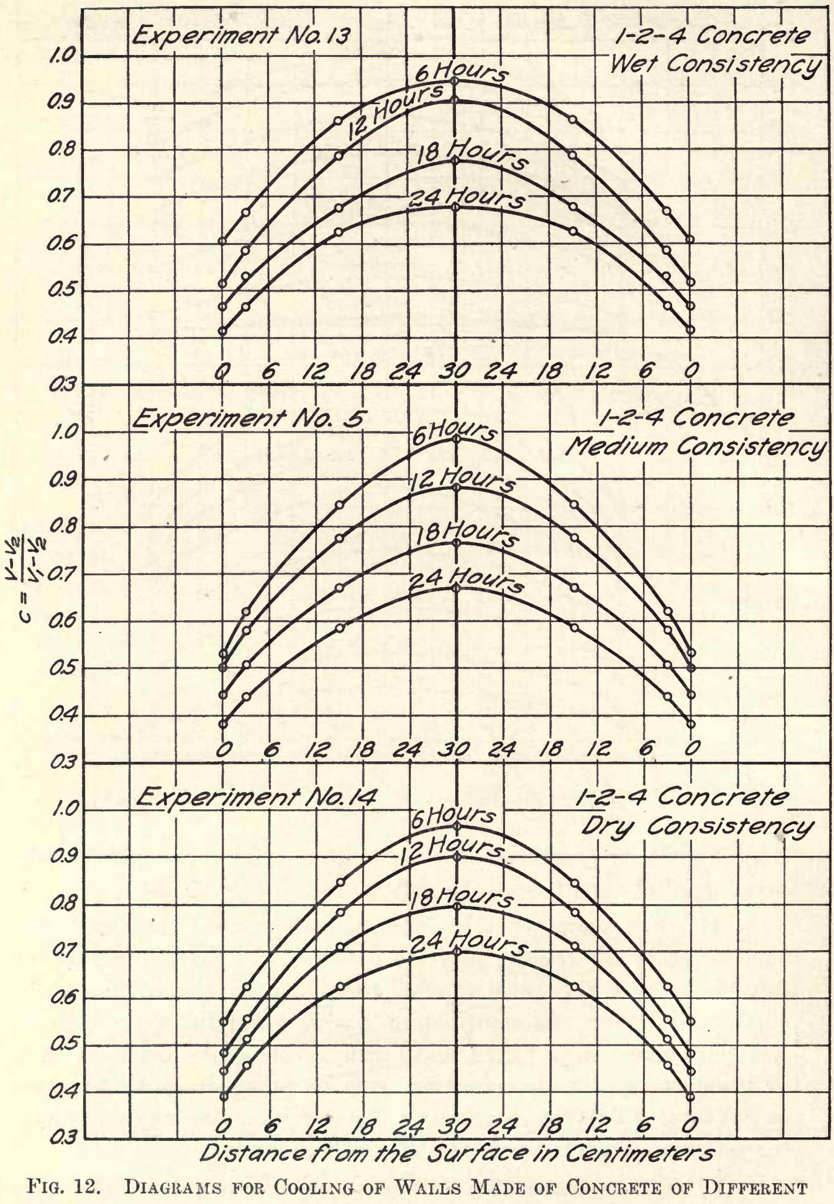

FIG. 12. DIAGRAMS FOR COOLING OF WALLS MADE OF CONCRETE OF DIFFERENT CONSISTENCIES

It is important that as much strength as possible be acquired at an early age, if the concrete is later to be subjected to low temperatures. It is, therefore, important and necessary in cold weather to protect concrete very carefully for at least twelve hours after pouring, because during that time the chemical action of cement and water is greatest, the increase in strength is very marked, the heat produced helps to retard the later cooling of the concrete, and protection during this period makes the concrete better able to resist the effect of low temperature.

(3) It is apparent from the analysis that very much more care must be taken to prevent the freezing of the concrete in relatively light structures than in massive work.

(4) Wind has very great effect on the cooling of fresh concrete. Every precaution should be taken to protect the surface from the wind.

III. Analytical Applications

16. *Massive Concrete with One Surface Exposed.*—Consider a mass of concrete of indefinite lateral extent and having one surface exposed to the air. Consider the temperature of the ground to be the same as that of the concrete when poured. For simplicity it is assumed that the soil has the same diffusivity as the fresh concrete, as would be approximately true in ordinary cases. The difference between the initial temperature of the concrete in place and the temperature of the air during the time considered (assumed to remain constant) will be called v_o, and the difference between the temperature of the air and that of any point in the concrete at any time t after pouring will be called v, the centigrade scale being used. Take the origin of distances at the surface of the concrete, and the positive x direction as running into the concrete.

Then the cooling is due to the radiation, convection, and evaporation of water at the surface (the constants determined from experiments taking care of evaporation). The equations for the temperature are:

$$\frac{\partial v}{\partial t} = h\frac{\partial^2 v}{\partial x^2} \qquad 0 < x < \infty$$

$$-\frac{\partial v}{\partial x} + hv = o \qquad \text{at} \quad x = 0$$

$$v = v_o \qquad \text{for} \quad t = 0$$

It is assumed that the temperature at infinity is constant and always equal to v_o, so that

$$v = v_o \qquad \text{for} \qquad t = 0$$

The solution* of these equations is

$$v = \frac{2v_o}{\sqrt{\pi}}\int_o^{\frac{x}{2\sqrt{kt}}} e^{-u^2} du + \frac{2v_o}{\sqrt{\pi}} e^{hx + h^2kt}\left(\int_o^{\infty} e^{-u^2} du - \int_o^{\frac{x+2hkt}{2\sqrt{kt}}} e^{-u^2} du\right)$$

$$\text{or } \frac{v}{v_o} = \theta\left(\frac{x}{2\sqrt{kt}}\right) + e^{hx + h_2kt}\left\{1 - \theta\left(\frac{x}{2\sqrt{kt}} + h\sqrt{kt}\right)\right\} \ldots\ldots (6)$$

* Carslaw, "Fourier's Series and Integrals," page 245, 1906.

where,

$$\theta(x) = \frac{2}{\sqrt{\pi}}\int_o^x e^{-u^2}\,du.$$

In the equation

$v =$ temperature in degrees centigrade,
$x =$ depth below the surface in cm.,
$t =$ time in seconds,

using the values $k = 0.0063$ and $h = 0.046$ which have been determined from the experiments, equation (6) becomes

$$\frac{v}{v_o} = \theta\left(\frac{x}{0.1588\sqrt{t}}\right) + e^{0.046x + 0.00001333t}\left\{1 - \theta\left(\frac{x}{0.1588\sqrt{t}} + 0.00365\sqrt{t}\right)\right\} \qquad (7)$$

From this equation, the temperature at any depth below the surface at any time after pouring the concrete may be obtained.

The temperature at the surface at a certain time after the concrete has been poured may be written by equation (6) as follows,

$$v = cv_o \qquad (8)$$

where

$$c = e^{0.048t}\left\{1 - \theta\,(0.219\sqrt{t})\right\}$$

t being measured in hours. The numerical value of c is less than unity.

It should be noted that for the constants used the equations given will hold good only until the concrete begins to freeze.

When the surface is protected with a board, it is convenient to assume a thickness of concrete equivalent in effect to the given thickness of board in the manner described in Article 12, using $h = 0.0031$ and $k = 0.0063$. Then equation (6) will apply to this case, the temperature at the depth of the equivalent thickness of concrete giving the temperature at the surface of the concrete against the board.

17. *Thin Wall or Slab.*—In a thin wall or slab having two surfaces exposed, cooling takes place from both sides. Call v_o the difference in temperature between the initial temperature of the concrete and the assumed temperature of the air, and v the difference in temperature between the air and any point in the concrete at a time t, the point being distant x from one of the surfaces.

Then equation (1)

$$\frac{v}{v_o} = 2\sum_{n=1}^{n=\infty} e^{-k\alpha_n^2 t} \frac{\alpha_n \cos\alpha_n x + h \sin\alpha_n x}{(\alpha_n^2 + h^2)\, l + 2h} \int_o^l (\alpha_n \cos\alpha_n x + h \sin\alpha_n x)\, dx$$

gives the temperature in the concrete in this case. The equation may be written as

$$v = cv_o$$

where c denotes the right hand member of equation (1).
In this equation

l = the thickness of the concrete in cm.
x = the distance from one of the surfaces to the point considered in cm.
t = time in seconds after pouring the concrete.

α_n = the $n\,th$ positive root of the equation

$$\tan \alpha l = \frac{2\,\alpha h}{\alpha^2 - h^2}$$

$h = 0.046$
$k = 0.0063$

When there are wood forms on one or both sides of the wall or slab, a thickness of concrete equivalent in effect to the given thickness of the board must be added to the thickness of concrete in the manner described in Article 12. To apply the equation, use the values $h =$ 0.031 and $k = 0.0063$ for the case of forms on both sides, and $h = 0.046$ and $k = 0.0063$ for the case with form on one side, to calculate the temperature of the unprotected side.

18. *Beam or Column of Rectangular Cross-section.*—If a thickness of concrete equivalent in effect to the given thickness of board is assumed as before, the following equation of flow of heat in a rectangular parallelepiped will give the temperature in a beam or column of rectangular cross-section with wood forms, at any time after pouring the concrete until the freezing of concrete begins. Let the initial temperature of a fresh concrete column or beam (a by b in cross section, c in length) be v_o above the temperature of the air, cooling taking place at the faces by radiation and convection into the air.

Then the following equations must be satisfied:

$$\frac{\partial v}{\partial t} = k\left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right\} \quad \text{in the solid;}$$

$$-\frac{\partial v}{\partial x} + h v = 0 \quad \text{at} \quad x = 0, y = 0, z = 0;$$

$$\frac{\partial v}{\partial x} + h v = 0 \quad \text{at} \quad x = a, y = b, z = c;$$

$$v = v_o \quad \text{at} \quad t = 0.$$

The solution* of these equations is

$$\frac{v}{v_o} = 4^3 h^3 \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} \frac{X_{2p+1}}{a(\alpha^2_{2p+1} + h^2) + 2h} \cdot \frac{Y_{2q+1}}{b(\beta^2_{2q+1} + h^2) + 2h} \cdot \frac{Z_{2r+1}}{c(\gamma^2_{2r+1} + h^2) + 2h} \cdot$$

$$e^{-k\left(\alpha^2_{2p+1} + \beta^2_{2q+1} + \gamma^2_{2r+1}\right)t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

or $v = c v_o$

where

$$X = \cos \alpha x + \frac{h}{\alpha} \sin \alpha x$$

$$Y = \cos \beta y + \frac{h}{\beta} \sin \beta y$$

$$Z = \cos \gamma z + \frac{h}{\gamma} \sin \gamma z$$

and α, β, γ are roots of the equations

$$\tan \alpha a = \frac{2 \alpha h}{\alpha^2 - h^2}$$

$$\tan \beta b = \frac{2 \beta h}{\beta^2 - h^2}$$

$$\tan \gamma c = \frac{2 \gamma h}{\gamma^2 - h^2}$$

Use C. G. S. units and $h = 0.031$ and $k = 0.0063$

19. *Column of Circular Cross-section.*—The temperature of the fresh concrete column of circular cross-section is v_o above the temperature of the surrounding air. Call l the radius of the column. It is assumed that both ends of the column are well protected and that cooling takes place through the convex surface of the column, so that the lines of flow of heat are radial in planes perpendicular to the axis of the column.

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* Carslaw, "Fourier's Series and Integrals," page 320, 1906.

Then the equations of temperature expressed in cylindrical co-ordinates, taking the center as origin, are

$$\frac{\partial v}{\partial t} = k\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right), \quad 0 < r < l$$

$$\frac{\partial v}{\partial r} + h v = 0 \qquad \text{at } r = l,$$

$$v = v_o \qquad \text{for} \quad t = 0.$$

The solution* of these equations is

$$\frac{v}{v_0} = \frac{2h}{l} \sum_1^{\infty} e^{-k\alpha_n^2 t} \cdot \frac{J_o(\alpha_n r)}{(\alpha_n^2 + h^2)\, J_o(\alpha_n l)} \qquad \text{.........(10)}$$

$$\text{or} \quad v = c\, v_o$$

in which $\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots$ are the roots of the equation

$$\alpha J_o'(\alpha l) - h J_o(\alpha l) = 0$$

and J_0 is the Bessel's function of the first kind of zero order. The values of the thermal constants h and k may be taken as $h = 0.046$ and $k = 0.0063$, C. G. S. units being used.

When there are wood forms, a thickness of concrete equivalent in effect to the given thickness of the board must be added to the thickness of concrete in the manner described in Article 12 to apply equation (10). Call l the total radius of the column including the additional thickness equivalent to the forms, and use the values $h = 0.031$ and $k = 0.0063$. Then equation (10) will give the temperature in the column under still air conditions at any point at any time until the freezing of the concrete begins.

* Carslaw, "Fourier's Series and Integrals," Page 313, 1906.

IV. Some Application of Experimental Data and Analysis to Work of Concreting in Freezing Weather

20. *General Considerations.*—Heat hastens the setting and hardening of concrete; cold delays it. The effect of cold becomes noticeable in this respect when the temperature falls below 50 deg. F. (10 deg. C.), and becomes more marked with lower temperatures. At temperatures below the freezing point of water the fresh concrete will freeze.

If the temperature of the concrete when placed is low, setting will be delayed, and, if subjected to further cooling by low air temperature, the concrete may freeze before it has attained sufficient strength to withstand the stresses to which it may be subjected, or even before it has taken its set. Serious objection should be made to allowing concrete to freeze before it has set or even before it has hardened very long. The general opinion is that a freezing temperature will not injure concrete that has first had an opportunity to harden for at least 48 hours under favorable conditions, though repeated freezing and thawing at an early age may be expected to cause injury. It is better and safer to protect concrete until freezing temperatures will have no injurious effect on it than to expose it to freezing at too early an age. Accordingly, the exposure of fresh concrete to freezing is not considered here, and the discussions that follow apply only until such a time as the freezing of fresh concrete would begin.

To predict the temperature of the concrete after a certain time by the application of the theory of heat conduction, it is necessary to forecast atmospheric temperature for some time after the concrete has been deposited. The forecast of the weather bureau will generally be found of great assistance.

In general, concrete work in winter is more difficult and somewhat more expensive than in summer; but policy frequently makes work in cold weather necessary and even economical. However, in order to attain satisfactory results in this case, proper precautions have to be observed.

It is the purpose of this chapter to describe some applications of the experimental data to concreting in freezing weather. It is not considered that the solutions given are very exact, but it is hoped to provide some information concerning the precautions to be observed.

The general effect of heating the materials, the presence of the forms and other protective covering, and the use of chemicals to lower the freezing point will be considered.

21. *Concrete with One Surface Exposed.*—The larger concrete constructions such as foundations, abutments, and retaining walls fairly well protected on the sides, may be considered to have the cooling take place by radiation and convection over one exposed surface, and the equations given in Article 16, "Massive Concrete with One Surface Exposed," may be considered to apply. The cooling from the sides of the structure will be small as compared with that from the exposed surface.

From equation (7) the diagrams in Figs. 13 and 14 have been prepared; v_1 is the initial temperature of the concrete; v_2 is the assumed air temperature to which the exposed surface will be subjected; v is the temperature attained by the concrete at a given distance from the surface at the given time after the concrete was poured. As $\frac{v-v_2}{v_1-v_2} = c$ is a ratio, any thermometric scale may be used, and the diagrams are, therefore, applicable to the Fahrenheit scale. It should be borne in mind that for the constants used, the diagrams do not apply after the concrete begins to freeze.

As an example, assume the temperature of the finished concrete to be 60 deg. F., and that of the air the following night 10 deg. F. From Fig. 13 or Fig. 14 the ratio c for temperature at the surface after 12 hours is

$$c = \frac{v-v_2}{v_1-v_2} = \frac{v-10}{60-10} = 0.5,$$

$$\therefore \qquad v = 35 \text{ deg. F.}$$

After 12 hours the temperature 4 in. below the surface (from Fig. 13 or Fig. 14, ratio $c = 0.705$) will be

$$v = v_2 + 0.705\ (v_1 - v_2) = 45 \text{ deg. F.}$$

To determine the time when the freezing point will be reached at the surface of the concrete for the temperatures just used, the ratio

$$\frac{v-v_2}{v_1-v_2} = \frac{32-10}{60-10} = 0.44, \text{ and}$$

from Fig. 13 the time corresponding to this ratio is 18 hours.

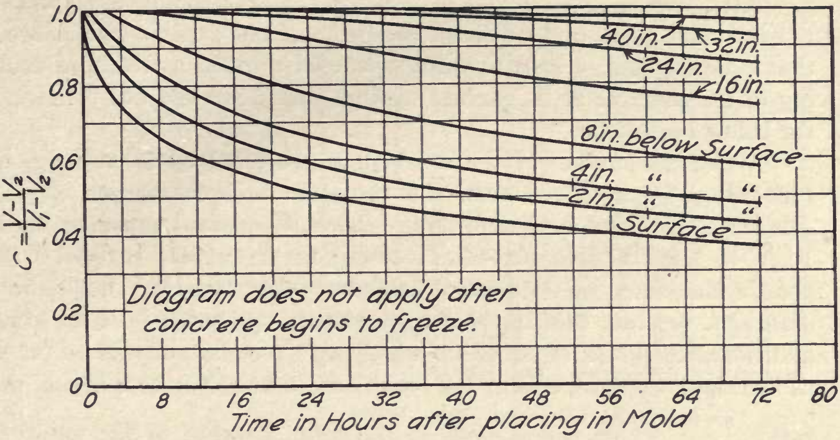

FIG. 13. DIAGRAMS FOR COOLING OF MASSIVE FOUNDATIONS

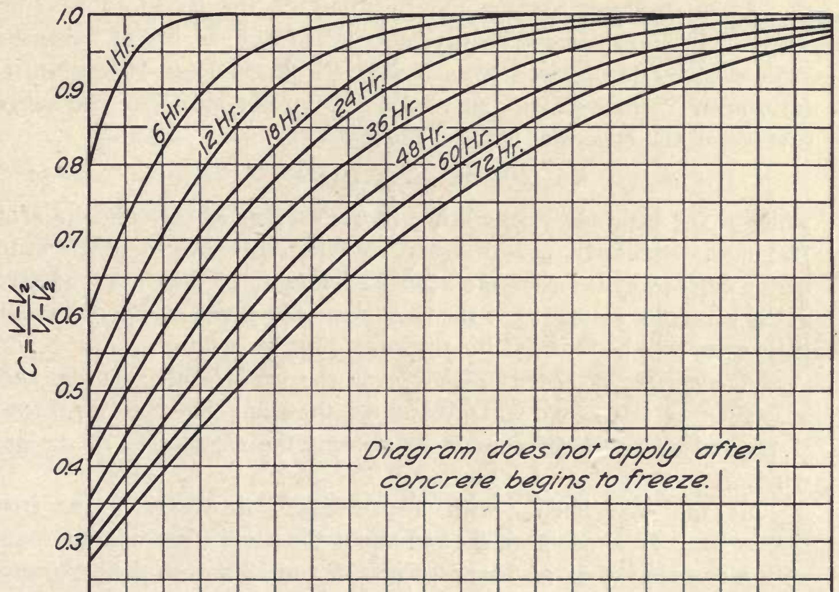

FIG. 14. DIAGRAMS FOR COOLING OF MASSIVE FOUNDATIONS

Attention is again called to the fact that the cooling is assumed to have taken place under still air conditions. The experiments showed that exposing the surface to wind produced a much more rapid cooling of the concrete at its surface, though the effects did not penetrate far below the surface.

When the surface of the concrete is protected by a covering, as for example a wooden form, the radiation and conduction at the surface is modified and a different thermal constant must be used. Figs. 15 and 16 give curves of cooling for a surface covered with boards, calculated by the method described in Article 16. To use these diagrams, consider that the board covering is equivalent in effect to an additional thickness of concrete, which may for the purpose be taken as five times the thickness of the board, and take from the diagram the ratio $\frac{v - v_2}{v_1 - v_2}$ which corresponds to the total distance to the point in question, including in this distance the equivalent thickness due to the thickness of the form.

As an example assume the thickness of the board to be 1 in. This will be considered equivalent to adding 5 in. of concrete. Take v_1 as 60 deg. F. and v_2 as 10 deg. F. From Figs. 15 and 16 the ratio after 72 hours at a depth of 5 in., corresponding to the actual surface of the concrete, is 0.5. Then

$$v = 10 + 0.5\ (60 - 10) = 35 \text{ deg. F.}$$

which is the expected temperature at the surface of the concrete after 72 hours. Similarly, a 2-in. board covering will show a temperature at the surface of the concrete after 72 hours of 41 deg. F. The foregoing examples assume that the thickness of the concrete is not small; little error will be found if the thickness is at least 6 in.

A covering of boards placed over the surface of concrete, such as a floor or pavement, will give much the same effect as the forms, if the covering is tight enough to prevent the circulation of air and wind.

As the experiments with a protection of canvas 3 in. from the surface of the concrete showed about the same effect as that found with a covering of ¾-in. boards, Figs. 15 and 16 may also be expected to give values of the ratio approximating the conditions of the canvas cover.

For a covering of earth Figs. 13 and 14 may be expected to approximate the cooling conditions, the effect of a given thickness of

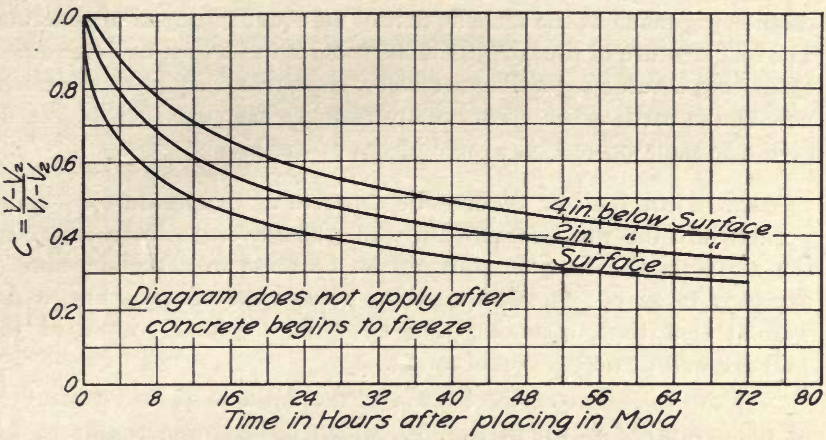

FIG. 15. DIAGRAMS FOR COOLING OF MASSIVE FOUNDATIONS PROTECTED BY WOOD FORMS

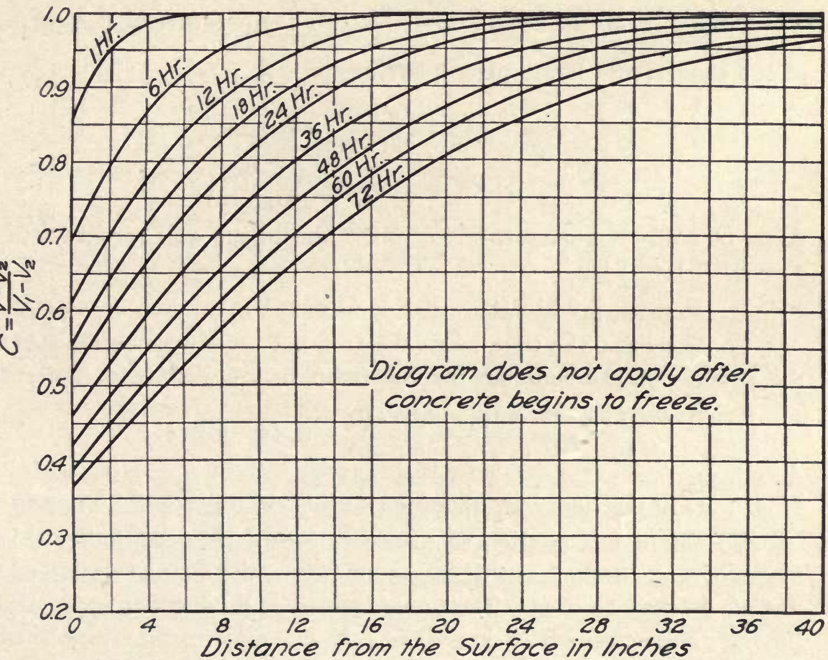

FIG. 16. DIAGRAMS FOR COOLING OF MASSIVE FOUNDATIONS PROTECTED BY WOOD FORMS

earth being taken as the same as that of an equal thickness of concrete. The temperature of the surface of the concrete will then be determined by finding a value for a distance below the surface equal to the thickness of the earth cover. This really assumes that one inch of boards gives the same protection as five inches of earth.

22. *Thin Wall or Slab.*—The concrete of temperature v_1 is deposited into the wood forms of a wall. The average temperature of the air from the time of pouring the concrete to a certain time is expected to be v_2. It is required to predict the temperature in the wall at that time, assuming that both the top and bottom of the wall are well protected from cooling.

If the thickness of the boards of the form is ¾ in., the curves of temperatures shown in Fig. 10, which embody the results of the experiments, are directly applicable for walls having the thicknesses of 24 in., 16 in., and 8 in. respectively. As an example, assume $v_1 = 60$ deg. F., $v_2 = 10$ deg. F., and thickness of the wall = 16 in.

From Fig. 10 the ratio $c = \frac{v - v_2}{v_1 - v_2}$, for temperature at the surface of the concrete 36 hours after pouring, is

$$c = \frac{v - v_2}{v_1 - v_2} = \frac{v - 10}{60 - 10} = 0.525$$

$$\therefore \qquad v = 36 \text{ deg. F.}$$

After 36 hours the temperature at the center of the wall (from Fig. 10, $c = 0.615$) will be

$$v = 10 + 0.615\ (60 - 10) = 41 \text{ deg. F.}$$

To determine the time when the freezing point will be reached at the surface of the concrete for the temperature just used, the ratio

$$c = \frac{v - v_2}{v_1 - v_2} = \frac{32 - 10}{60 - 10} = 0.44, \text{ and}$$

from Fig. 10 the time corresponding to this ratio is about 46 hours.

To obtain information as to the rate of cooling for other thicknesses of walls and other thicknesses of forms, the method described in Article 12 may be used. The temperatures at different times in walls having thicknesses of 16 in., 20 in., 24 in., and 32 in., respectively, have been calculated in the manner described in Article 17 and plotted in Figs. 17 and 18. To use these diagrams, consider that

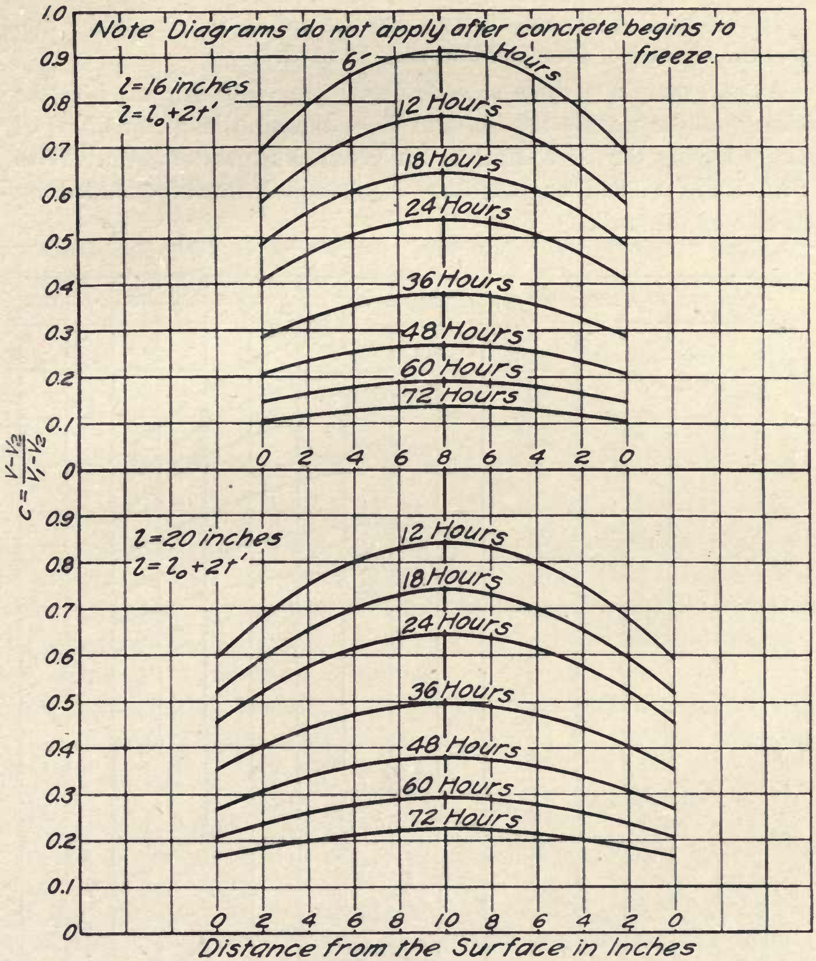

Fig. 17. Diagrams for Cooling of Walls Protected by Wood Forms

the boards of the forms are equivalent in effect to an additional thickness of concrete, which may for the purpose be taken as five times the thickness of the board, and calculate the total thickness of the wall. From the diagram corresponding to the total thickness, take the ratio $\frac{v-v_2}{v_1-v_2}$ which corresponds to the total distance to the point in question, including in this distance the equivalent thickness of concrete corresponding to the thickness of the form.

If this total thickness lies between the values shown on the diagrams the method of interpolation may be used.

As an example, assume $v_1 = 60$ deg. F., $v_2 = 10$ deg. F., $l_0 =$ the thickness of the wall = 14 in., and $t' =$ the additional thickness of concrete having the same effect as the given thickness of form. With a 1-in. board t' will be taken as 5 in., which must be added on each side of the wall.

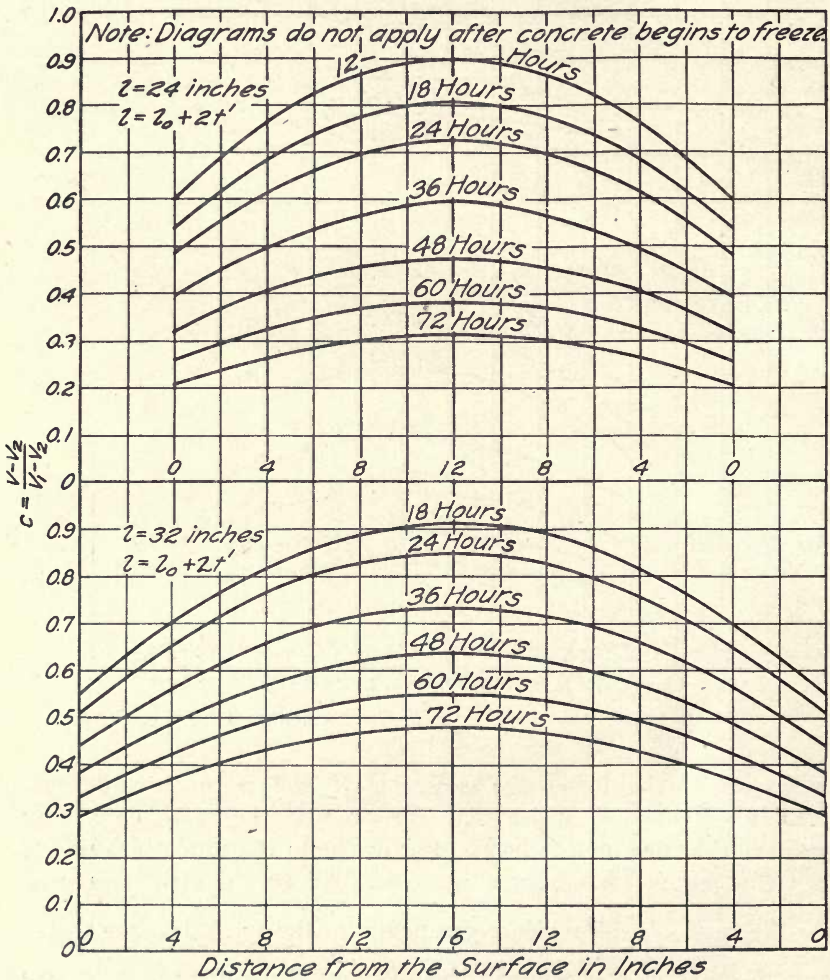

FIG. 18. DIAGRAMS FOR COOLING OF WALLS PROTECTED BY WOOD FORMS

Then the total thickness of the wall is

$$l = l_0 + 2\,t' = 14 + 10 = 24 \text{ in.}$$

From Fig. 18 the ratio $\frac{v - v_2}{v_1 - v_2}$ after 36 hours, at a depth of 5 in., corresponding to the actual surface of the concrete, is 0.52. Then

$$v = 10 + 0.52\,(60 - 10) = 36 \text{ deg. F.}$$

Similarly, the time at which the freezing temperature will reach the concrete is estimated to be 45 hours.

23. *Column of Circular Cross-section.*—Consider that concrete of temperature v_1 is deposited into wood forms of circular cross-section, the temperature of the surrounding air being v_2. Assuming the equivalent thickness of concrete for the board of the form, let the total radius of the equivalent column be

$$l = r_0 + t'$$

where r_0 is the radius of the column and t' is the additional thickness of concrete having the same effect as the given thickness of form, which may for the purpose be taken as five times the thickness of the board. If both ends of the column are well protected from cooling, equation (10) applies to this case.

For the values of radius, l, of 10 in., 14 in., and 20 in., respectively, the ratio $c = \frac{v - v_2}{v_1 - v_2}$ has been calculated by equation (10) in the manner described in Article 19 and plotted in Fig. 19.

The use of this figure is similar to that of Figs. 17 and 18.

24. *Heating the Materials.*—A good method to use in concreting in freezing weather is to heat the materials. Frozen aggregate is frequently considered to be the cause of failure of concrete laid in freezing weather. The water, the sand and the water, or the entire aggregate and the water should be heated, as called for by circumstances.

It is clear that heating the materials lengthens the time before the mixture becomes cold enough to freeze. In order that the freezing temperature may not penetrate into the surface of fresh concrete it is necessary that, using Fahrenheit units,

$$v_1 > \frac{32}{c} - \frac{(1-c)}{c} v_2 \qquad \text{(11)}$$

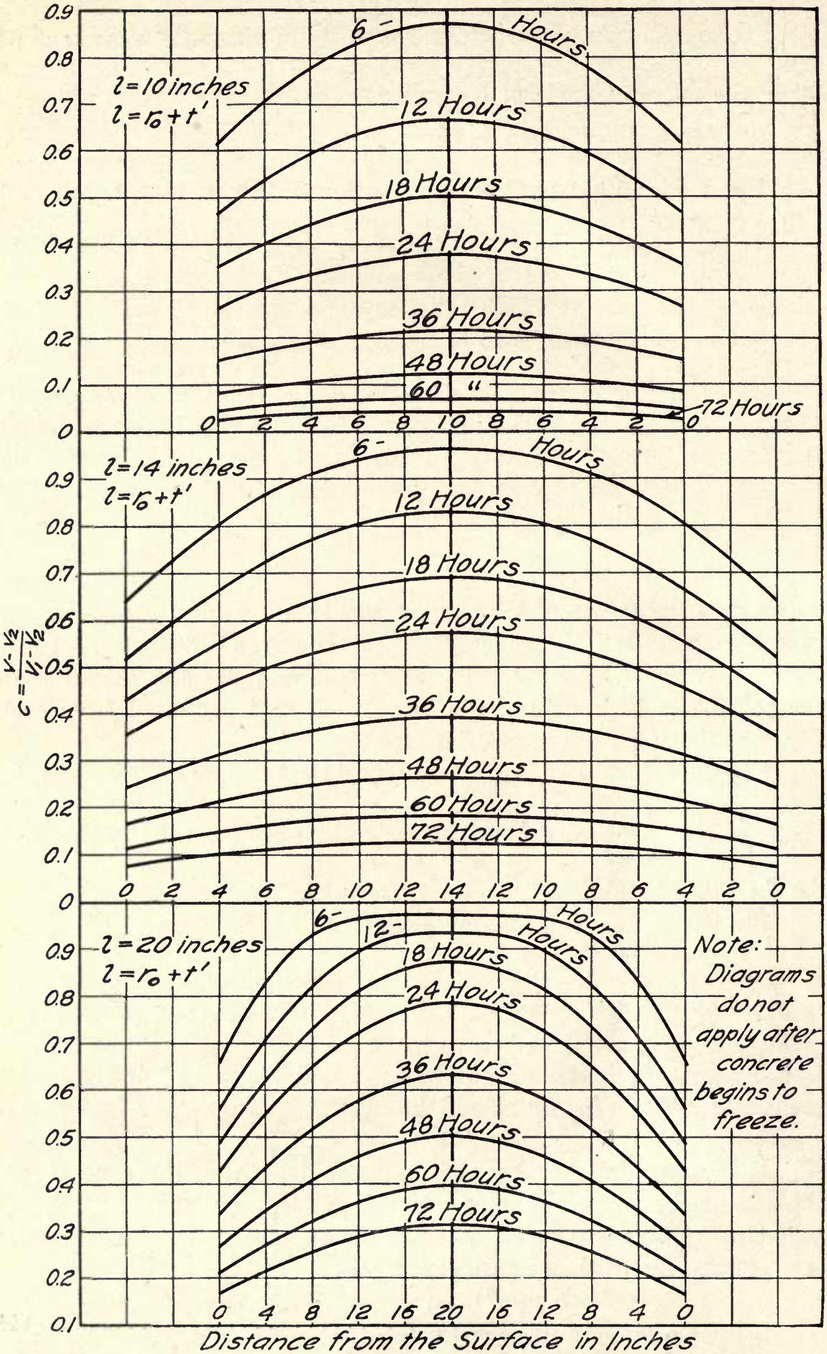

Fig. 19. Diagrams for Cooling of Circular Columns Protected by Wood Forms

where c is the ratio $\frac{v - v_2}{v_1 - v_2}$ at the surface of the concrete at a given time after pouring.

This expression gives the temperature at which the concrete must be finished to prevent its freezing after it has been exposed to the low temperature for the given length of time. The values of c for foundations, walls, etc., may be obtained from the diagrams for the several cases already explained.

As an example, in order that the freezing temperature may not penetrate below the surface of the concrete foundation, used as an example in Article 21, after 48 hours with the temperature of the air 10 deg. F., the temperature at which the concrete must be placed is found from Fig. 13 and formula (11) to be 79 deg. F.

If, in the same example, the fresh concrete is deposited at 100 deg. F. instead of at 60 deg. F., the temperature used in the example in Article 21, the time required for the surface of the concrete to cool to 32 deg. F. will be found by extrapolation to be about 80 hours instead of 18 hours.

There is a practical limit to the value of v_1, as excessive heating may accelerate too much the setting of the cement. The safe maximum temperature of the fresh concrete may be taken as about 100 deg. to 120 deg. F. Putting these values in formula (11), it becomes

$$v_2 > \left(\frac{32 - c\,(100 \text{ to } 120)}{(1 - c)} \right) \qquad (12)$$

For a given length of time after pouring, the lowest expected temperature, when using this method of heating the materials, can be obtained from the above expression.

It is to be noted here, that, as heating the materials accelerates the rate of hardening, this also helps to insure the setting of the concrete before it can be damaged by frost.

In using this method it should be kept in mind that every effort must be taken to keep the fresh concrete at a temperature not below the specified one. The rapidity with which the work can be done after the heating is stopped, needs much consideration. As a practical precaution special care must be taken to remove all frozen lumps of aggregate and all ice and snow crystals.

25. *Lowering the Freezing Point of the Mixing Water by Using Chemicals.*—One way of preventing the freezing of fresh concrete in cold weather is to lower the freezing point of the water in the con-

crete by employing chemicals, usually common salt, or sodium chloride, and to a lesser extent calcium chloride.

Though this method is effective only for temperatures a little below freezing, and should not be used where there is danger of electrolysis, nor in concrete reinforced with steel, it is sometimes used because of the simplicity of the process.

Approximately, by Professor Tetmajer's rule reduced to Fahrenheit units, each per cent by weight of salt added to the water reduces the freezing point by one degree Fahrenheit, no more than 10 per cent by weight being considered safe practice.

Formula (11) is applicable to this case. Using 10 per cent salt brine, the relation between v_1 and v_2 is

$$v_1 > \frac{22}{c} - \frac{(1-c)}{c} v_2 \qquad (13)$$

As may be seen from the curves of cooling for the several cases, in ordinary cold weather the freezing temperature will not penetrate many inches into the concrete in two or three days after pouring. Consequently, it is not necessary to use brine with the whole mass of concrete. Only the surface layer of the concrete need be made with brine.

If the temperature of the air is much below the freezing point of the mixture, equations (11) and (13) show that the advantage of the chemical is small. In fact, protection by some form of covering is generally much to be preferred, especially as the rate of hardening is reduced both by the presence of the salt and the lower temperature attained.

26. *General Conclusions.*—It is believed that the constants derived from the tests are applicable to the ordinary conditions of fresh concrete. The value of k, the diffusivity, is 0.0063; that of h, the ratio of the emissivity to the coefficient of conductivity, is 0.046.

From the cooling curves given for massive concrete, thin walls and slabs, and beams and columns, the time of cooling and the temperature attained by the concrete in any particular case may be estimated.

The protection afforded by board forms and by canvas is considerable, as was shown by the examples given.

If a canvas protection is used, care must be taken to prevent any circulation of cold air, and it is best to observe the temperature in the

air space by means of a thermometer. Knowing this temperature, the time when the freezing temperature will penetrate into the concrete may be estimated from the diagrams and the necessity of using artificial heat determined.

The heating of the materials is seen to furnish an excellent method of ensuring early hardening and delaying the fall in temperature. It is much to be preferred to the use of chemicals.

The effect of the wind should also be given consideration.

The diagrams are of some value in connection with the determination of the time for the removal of the forms, since they will enable the temperature attained by the concrete to be estimated. As the concrete gains strength much more slowly at a low temperature, and as frozen concrete is especially dangerous, information on the temperature attained is of value and the too early removal of forms may be avoided. If by this or other means the mean temperature of the concrete for a given time is known, the comparative strength of the concrete may be estimated from the diagrams given by Professor A. B. McDaniel in Bulletin No. 81 of the Engineering Experiment Station of the University of Illinois, which shows the influence of temperature on the strength of concrete, and thus the judgment may be aided in deciding on the safe time to remove the forms.

LIST OF
PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

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Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

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